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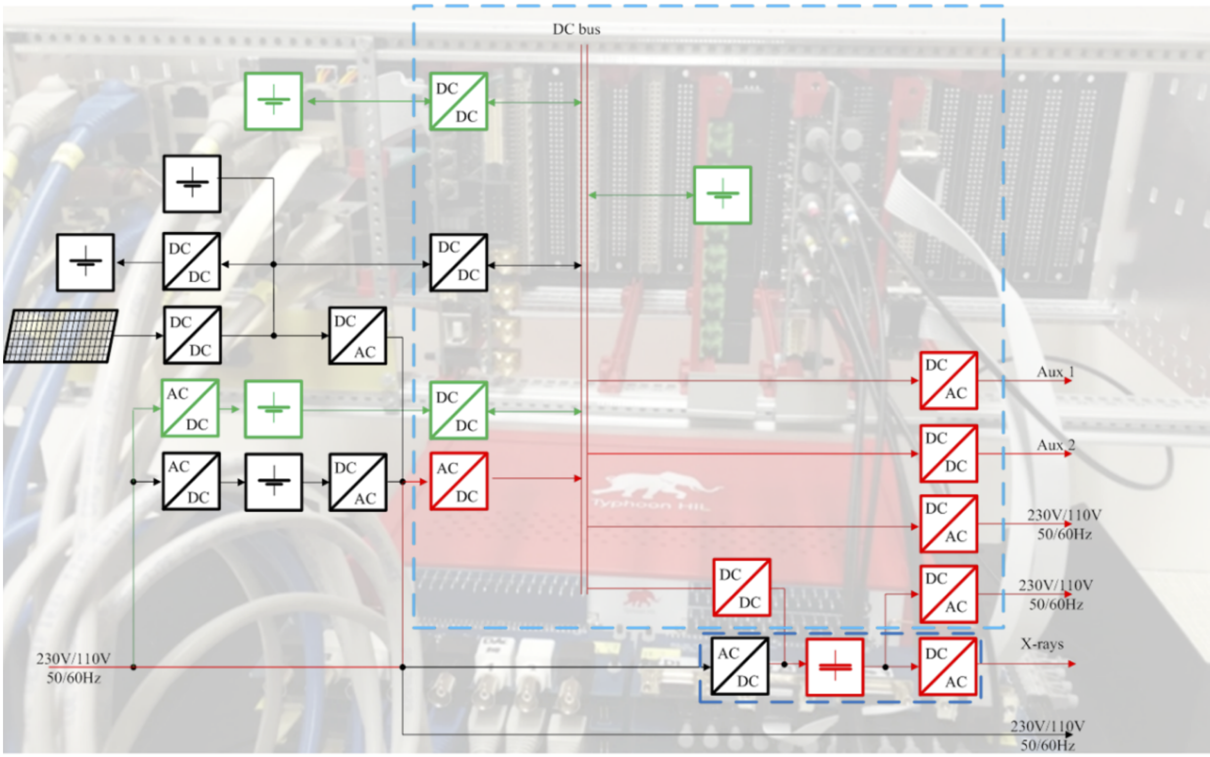
Department of the Environment,
Transport, Energy and Communication DETEC

Swiss Federal Office of Energy SFOE
Energy Research

Final report

Electrical stability of DC and hybrid AC/DC, converter-based microgrids
State-of-the-art, open questions and challenges



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Date: November 2, 2021

Place: Bern

Publisher:
Swiss Federal Office of Energy SFOE
Research Programme, Energy - Economy - Society (EES)
CH-3003 Bern
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SFOE contract number: SI/502071-01

The author of this report bears the entire responsibility for the content and for the conclusions drawn therefrom.

Summary

Modern electric power grids are relying more and more on AC/DC and DC/DC electrical current conversion. Converters are in particular needed to connect components such as wind turbines and photovoltaic panels, electric battery storage and so forth to AC power grids. Converters present a number of advantages, however they often impact electrical energy efficiency negatively. Deploying DC microgrids reduces the number of conversions, thereby enhancing energy efficiency and reducing infrastructure costs, however it may simultaneously generate electrical instabilities. Especially strong such instabilities are expected to emerge as microgrids become larger and larger, with the addition of more and more components. The goal of this project is to identify a focused research direction on electrical stability of hybrid AC/DC grids, to identifiy the related key issues and central problems and to formulate a multi-pronged strategy based on numerical simulations, hardware-in-the-loop and other analog emulations to guarantee the stability of future hybrid AC/DC grids. This report presents the state-of-the-art, identifies the main challenges and provides a list of the main questions to address, in order to facilitate a smooth deployment of evolving DC microgrids externally connected to an AC grid.

Résumé

De nombreux composants électriques, tels que panneaux photovoltaïques, batteries de stockage électrique, éoliennes etc. sont connectés au réseau électrique par des convertisseurs de courant AC/DC ou DC/DC. Pour réduire le nombre de conversions, et donc leur coût, tout en améliorant l'efficience énergétique du réseau, le déploiement de micro-réseaux DC a été proposé. Ces installations peuvent néanmoins devenir le lieu d'instabilités électriques, qui peuvent être graves, surtout lorsque la taille de ces réseaux augmente par l'ajout de composants. Le but de ce projet est d'identifier une ligne de recherche focalisée sur la stabilité électrique des réseaux hybrides AC/DC, d'identifier les questions et problèmes principaux et de formuler une stratégie basée sur des simulations numériques et des émulations de type hardware-in-the-loop pour favoriser la stabilité des réseaux hybrides AC/DC futurs. Ce rapport résume l'état de l'art, identifie les problèmes principaux et liste les questions principales à aborder pour faciliter le déploiement de micro-réseaux DC stables, connectés extérieurement à un réseau AC.

Contents

1 Introduction

In the developed world, electric power is almost exclusively distributed via large AC power grids. There are historical reasons for that which are (i) until recently, electric power generation relied almost exclusively on large, decentralized plants, located often far away from consumption centers and (ii) electrical grid stability and safety of supply - in particular keeping the balance between production and consumption - is generally improved by connecting regional grids together, thereby mutualizing their resources in production and ancillary services. Energy efficient transmission of electric power over large distances then requires efficient transformation between low voltages (at which power is produced and consumed) to high and very high voltages (at which it can be transmitted with minimal ohmic losses). Voltage transformation is easily and efficiently achieved for AC currents.

Important changes have been strongly impacting the just described electric transmission and distribution model. The energy transition in the electricity sector induces a steady transition towards new renewable sources of energy, in particular photovoltaic panels and wind turbines which are DC in nature and are therefore connected to the AC grid via converters. They are furthermore more decentralized than traditional power plants, i.e. more likely to be closer to consumers, which in principle reduces the need for energy-efficient, long-range power transmission. Alternative solutions complementing the AC grid have been explored in that context, one of them being low voltage DC microgrids.

There are two main motivations for studying and deploying low voltage microgrids. First, communities exist in rural/isolated areas, where it makes more sense financially and/or logistically to develop an autonomous microgrid than to extend the existing AC power grid. Energy access is a key ingredient to reduce poverty and microgrids have been recognized as a way to reduce the *energy access gap* [1]. This aspect is of course more important in developing countries than in Switzerland, where communities still exist in remote areas that could be deserved by such islanded microgrids. The question then arises as to whether an AC or a DC grid is more appropriate. Assessments of AC and DC on the basis of a number of criteria (existing generation and load types, electric safety and stability, size, components, deployment and maintenance cost and so forth) is presented in Ref. [2, 3]. In many instances, DC microgrids are the solution of choice to connect inherently DC productions such as photovoltaic panels or wind turbines and electric batteries - also inherently DC - because progresses in power electronics has rendered many loads also DC in nature [3]. The second main motivation to study low voltage microgrids is therefore that, in their DC configuration, they allow one to connect inverter-connected components (batteries, photovoltaic panels) with a reduced number of conversions. DC microgrids are therefore often less expensive and more energy efficient. It is also noticeable that both AC and DC microgrids require voltage stabilization, however, AC grids additionally require frequency stabilization. A priori, DC microgrids are therefore less likely to suffer from instabilities. To make a long story short, the energy transition calls for an increased penetration of new renewable sources of electrical power, and DC microgrids allow to incorporate these local, inherently DC sources more efficiently, either into a local, autonomous microgrid or a microgrid externally connected to the AC distribution/transmission grid. This is the motivation behind the thematic developed in this report.

These advantages do not mean, however, that DC microgrids are free of instabilities. It has long been known in particular that instabilities occur in DC microgrids when converters are connected with a setpoint such that they absorb a constant power. Such a setpoint is actually the rule more than the exception, in particular because loads vary over time scales much longer than typical microgrid time scales - they are effectively constant from the point of view of grid dynamics. At a qualitative level, the instability occurs because an effective negative impedance $Z = \delta V/\delta I = -V/I$ corresponds to a constant power $P = IV$, $\delta P = I\delta V + V\delta I = 0$. Accordingly, each new converter-connected device has the potential to destabilize a formerly stable DC microgrid. Stabilization may in principle be achieved via the introduction of line resistances, resulting in positive damping terms in the corresponding differential equations. However, the associated ohmic dissipation reduces the microgrid energy efficiency and therefore, other, energy-optimized solutions are highly desirable. Such solutions are in principle case-dependent : each microgrid, with its specific configuration (including number of converters and their characteristic, line admittances, filter parameters and so forth) has its own stability characteristics that need to be investigated and that are in principle different from any other microgrid with a different geometry. Yet, one may wonder if some general rules or set of rules may be expressed, which would guarantee that adding one or few components to an initially stable microgrid would not jeopardize its stability. This is the focus of the present document, which will discuss the existing literature on the topic of stability designs for DC microgrids, identify gaps in the literature and list the main current challenges to the large-scale deployment of DC microgrid.

2 Goal of Project

The goal of the project is to summarize the state-of-the-art and identify the main research gaps in the topic of stability of DC microgrids, either islanded, or connected to an external AC power grid. A list of open problems, and main challenges to solve towards the safe deployment of microgrids is given below in this report. The problem is formulated an discussed in Section 3, where we further present the state-of-the-art and list the first questions to be addressed by future research and development projects on DC microgrids.

3 Formulation of the Problem

3.1 Historical and more recent milestones

It was recognized at least as early as 1973 (and perhaps even earlier) that converters regulated to absorb a constant load may generate voltage instabilities in DC grids [4]. These instabilities occur because connecting such regulated converters to DC grids requires to use filters – usually modeled by parallel capacitors – to damp the converter's high switching frequencies. In the presence of voltage and current variations, line inductions can furthermore no longer be neglected, even in DC systems. The presence of filters and constant-load converters, together with inductive effects may then lead to negatively damped voltage oscillations.

One of the main points brought about by the simple, few-bus analysis of Ref. [4] is that, even in a DC grid, line inductions cannot be neglected when considering voltage stability. This is so because when voltages start to fluctuate, so do currents, hence voltage stability is inevitably coupled to line inductances. This was recognized early by the power electronics community, which investigated the role of inductance vs. capacitive filters in DC microgrids [5, 6, 7, 8, 9, 10]. Some of these works proposed resistive-capacitive stability cells, put in parallel between filters and converters, others investigated the design of stabilizing input filters while still others specified impedances that guarantee stability in specific, few-bus DC microgrids. Still, until recently, many investigations of DC microgrids in the electrical engineering community considered purely resistive lines [11, 12, 13, 14, 15]. These approaches correctly give fixed-point solutions to Eq. (3) below, deriving, for instance, conditions for all of them to lie inside a desirable set of operating values (in this respect see in particular Ref. [14]). These solutions are in principle equilibrium solutions to the problem. However, they are useless in practice if they are unstable, i.e. if the dynamics governed by Eq. (3) has exponentially increasing solutions with time, as is the case in the presence of negative damping. Ref. [5] showed that stability can be restored through the addition of a stabilizing circuit in the form of a dissipative (resistive) component in parallel to the filter. To guarantee that stabilization does not occur at the cost of energy efficiency, a capacitor is added in series to that resistance, whose goal is to allow current to flow through the resistance only when voltages start to oscillate. Dissipation then sets in only when voltage oscillation damping is needed.

Most of these works investigate relatively simple, few-bus systems. They emphasize voltage stability issues and provide a qualitative understanding about why instabilities occur in DC grids with converters absorbing constant loads. It is desirable to extend them to larger grids, with different topologies that furthermore evolve as new components are connected to them. A key question is then :

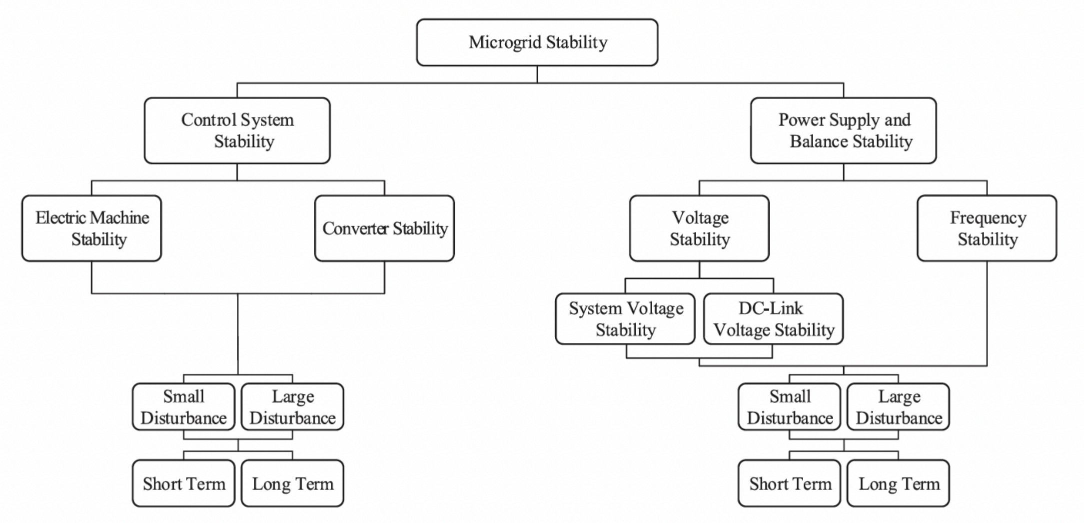

Figure 1: Classification of stability in microgrids. Figure taken from Ref. [16].

How can one guarantee that adding a new component to a previously stable DC grid is not jeopardizing its stability ?

To answer that questions several aspects have to be taken into account. In particular, it is important to keep in mind that microgrids are never globally stable. Excursion far away from anticipated setpoints will eventually destabilize the system. Therefore answering the above question significantly depends on how far from some expected operational state we want to ensure that the system remains stable. This, in its turn, requires that one considers various types/definitions of stability. Ref. [16] surveys microgrid stability definitions and gives a map of various stability issues In all instances, the stability problem is standardly formulated through a set of differential equations coupled over the DC grid, i.e. an undirected, weighted graph.

A classification map of stability definitions and/or issues is given in Fig. 1, taken from Ref. [16]. A first subclassification differentiates control system stability on the one hand from power supply and balance stability on the other hand,. They correspond, to whether stability is achieved by/instability occurs in active (controlled power electronics) components or passive (R, L and/or C) components. Cases of interest for the present project include converter stability in the first case, and voltage stability in the second. Next comes a second subdivision of the problem into stability under small or large disturbances, depending on whether one is satisfied with a treatment restricted to perturbations that are small enough that an analysis based on the linearization of the differential equations is sufficient - or not. In microgrids, each load represents a non-negligible fraction of the total load and accordingly, a small signal analysis based on linearized differential equations is often not sufficient. Below, we therefore put more emphasis on stability under large disturbances. It is interesting to note, however, that small-signal stability can be investigated via the Hurwitz criterion [17]. The latter relates stability to the positive-definiteness of a matrix, which in its turn is determined by the positivity of all the matrix' principal minor. Accordingly, the stability of a microgrid under the addition of few components to a previously stable microgrid requires to compute only few additional minors. We will discuss this further below. This approach, because of its apparent simplicity, will be listed below as one of the first proposed steps in an investigative program on the stability of DC microgrids.

3.2 Two-bus model

We illustrate the problem on a simple, two-bus model of a DC grid. This willl help us to identify the core origin of instabilities in DC grids with constant-load inverters.

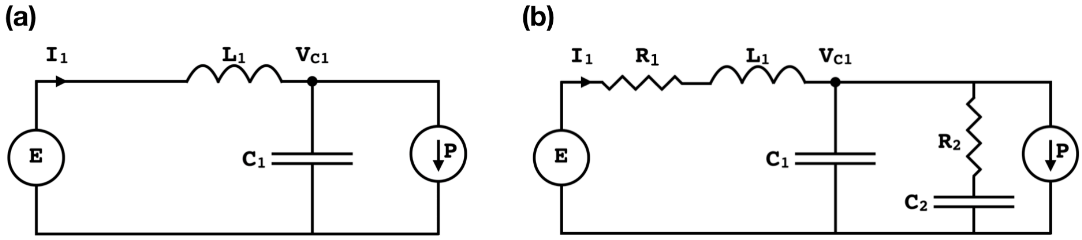

Figure 2: Converter absorbing a load P connected to (a) a purely inductive DC grid with a parallel capacitor acting as a frequency filter and (b) a resistive-inductive DC grid with a frequency filter and a resistive-capacitive stabilizing cell.

Let us consider a converter absorbing a constant load, connected to a DC grid fed by a voltage source. In the problem as formulated below, all time scales are shorter than typical load power fluctuations so that constant loads are the rule more than the exception. Connecting electric components to DC grids via regulated converters requires to use filters. The latter have a two-pronged task. First, they need to damp potential voltage fluctuations in the DC grid towards the converters and second, they need to suppress high frequency current fluctuations generated by the converter towards the DC grid. The filter is usually a capacitor installed in parallel to the load, with a capacity such that the filter frequency is much smaller than the converter switching frequency. It has been recognized that such a system may be subjected to voltage instabilities. Ref. [4] noticed that constant power loads result in negative admittances. This is easily understood by considering the two-bus system shown in Fig. 2(a), whose dynamics is determined by differential equations obtained from Kirchhhoff's laws (∂_t indicates derivatives with respect to time)

$$L_1 \partial_t I_1 + V_{C_1} = E \,, \tag{1a}$$

$$C_1 \partial_t V_{C_1} + \frac{P}{V_{C_1}} = I_1 \,. \tag{1b}$$

Substituting I_1 in Eq. (1a) from Eq. (1b) leads to

$$L_1 C_1 \partial_t^2 V_{C_1} - L_1 \frac{P}{V_{C_1}^2} \partial_t V_{C_1} + V_{C_1} = E - \frac{1}{V_{C_1}} \partial_t P \,. \tag{2}$$

This linear, inhomogeneous differential equation describes the dynamics of a damped, driven oscillator: the second term on the left-hand side gives the damping and the right-hand side gives the driving. The damping is clearly negative, which in principle can be compensated by the driving term. However, when the power absorbed by the converter is constant, $dP/dt = 0$, there is only a constant driving E which cannot compensate for the negative damping term. The general solution of Eq. (2) is given by the sum of the general solution to the corresponding homogeneous equation (with zero on the right-hand side) and a particular solution of the non-homogeneous equation. In particular the general solution to the homogeneous equation corresponding to Eq. (2) behaves as $V_{C_1}^{(0)} \exp[\lambda t]$ with $\lambda = P/(2C_1 V_{C_1}^2) \pm \sqrt{P^2/(2C_1 V_{C_1}^2)^2 - 1}$, which has a positive real part, corresponding to an exponentially increasing voltage V_{C_1}. Hence, there is an exponential instability of the fixed-point solution of Eq. (2) for constant load P.

This instability problem is described in detail in many publications, see e.g. Refs. [4, 5, 18, 19, 20], where a resistance R_1 is often added in series to the inductance L_1. Eq. (2) for constant load P is then modified to

$$L_1 C_1 \partial_t^2 V_{C_1} + \left(RC_1 - L_1 \frac{P}{V_{C_1}^2} \right) \partial_t V_{C_1} + V_{C_1} + \frac{PR}{V_{C_1}} = E \,. \tag{3}$$

A positive contribution RC_1 is now added to the damping coefficient, which makes it positive if $RC_1 - L_1 P/V_{C_1}^2 > 0$. When this is the case, the system is dynamically stable again – previously exponentially growing solutions now decay exponentially. It is important to realize, however, that this positive damping comes at the cost of a relatively high line resistance, i.e. relatively high ohmic losses. We would like to guarantee the system's stability without jeopardizing its energy efficiency.

Stabilizing the system while keeping ohmic losses low can be achieved by adding a resistance R_2 in parallel with the filtering capacitor C_1. If one additionally adds a capacitor C_2 in series with R_2, one can control the power dissipated through R_2 so that losses occur only when the voltage V_{C_1} across the filter $L_1 - C_1$ oscillates, i.e. only when there is a need for damping. This is described in Ref. [5], and the situation is sketched in Fig. 2(b).

3.3 Active components

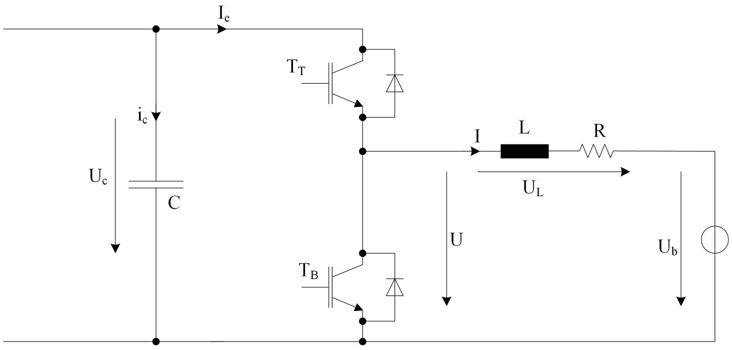

Figure 3: DC/DC converter for battery charge

The solution proposed in Fig. 2(b) is based on the addition of a damping resistor in parallel to the load. Resulting losses are lowered thanks to a series capacitor with well chosen capacity. Apart considerations on modeling and equations solving, this can be seen as the addition of a sub-circuit that degrades the "constant power" property and then leads to a stable solution. This damping circuit operates only when oscillations occur on the DC bus - with no time-dependence, there is no current flowing through the damping resistor thanks to the series capacitor.

This passive solution must be contrasted against active stabilization solutions, where the degradation of the "constant power" property is obtained via control of the power converters plugged on the DC bus. Such active solutions for a DC/DC converter for electric battery charge are illustrated in Figure 3. The input capacitance C is connected to the left to a DC grid (not shown in the figure). To determine the control process in such a converter, one first models it and then applies inversion rules to obtain a control scheme. This is schematically represented in Figure 4. Two frames can be seen in this figure. The right one represents an average model of the system presented in Figure 3. The left frame is the control defined by inversion rules. The DC stage "constant power" property is generated by (i) the controller bandwidth as usually designed, and (ii) the feed-forward term which rejects any DC bus voltage fluctuations. When instabilities arise, one can then act either on the current control bandwidth or more preferably on the way the feed-forward on voltage U_c is implemented. The latter solution is similar in behavior to the passive damping circuit solution of Fig. 2(b), with however the advantage that damping occurs without ohmic losses. Simultaneously, this solution limits the performances of the converter controls, with a redution of global performances for the end user.

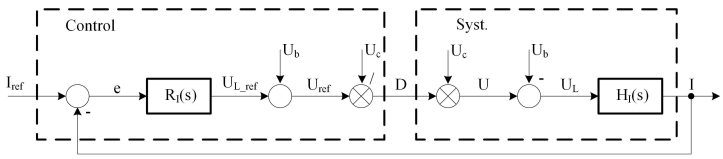

Figure 4: Inverse control scheme for a DC/DC converter

Such a stabilizing approach must be investigated because it can cause instabilities in the DC grid. The problem must be considered similarly (perhaps even simultaneously) as the passive approaches discussed above. Investigations along those lines have already been done to some extent in the literature.

Some solutions are proposed in Ref. [21], where a global supervision is proposed at the DC bus level. This supervision acts on each connected converter to ensure a global stability at the DC bus level. It must however be adapted at each change of the DC bus topology, and no general recommendation is provided for taking care of this adaptation. From a mathematical point of view, the approach is similar to the passive approach as global equations are similar. Standard tools are used for a small-signal stability analysis in a first approach, and large signal consideration are operated using Lyapunov theory [22].

Note that instabilities of grids due to negative impedances is not specific to DC networks. Similar issues also arise in AC grids, due to efficient (from the point of view of the end-user) control of converters [23]. One of the main conclusion is that part of the solution for limiting the impact of negative impedances on a grid is to act on the converters control bandwidth. This paper underlines that modeling an AC grid thanks to Clark/Park transform [24, 25] leads to the analysis of AC grids as an equivalent DC system. One must then identify how tools for the active stabilization of AC micro-grid translate into tools for DC grids.

Regardless of the chosen stabilization technique, it is important to realize that the general requirements for the definition of such networks do not include conditions for stabilizing their dynamics, even if requirements on control are specified or studied [26, 27]. Instead, what is generally specified is the dynamic control of DC grids from a pure system energy management point of view, aiming at controlling power flows between the various power sources plugged on such a grid. The dynamic property linked to stability and damping of a system are generally not taken into account [28, 29, 30, 31]

3.4 Stability criteria

We mentioned above that the voltage stability problem we focus on here is, roughly speaking, of either of two types – small signal and large signal stability, depending on whether one assumes that a linearization of the differential equations close to the operational fixed point is sufficient, or not.

The small signal case has, not unexpectedly, been considered in many articles. Once linearized close to a fixed-point of interest, systems of differential equations such as the ones in Eqs. (1)–(3) can be formally written as

$$\partial_t \mathbf{x} = \mathbf{A}\mathbf{x} + \mathbf{b}\,. \tag{4}$$

Here, $\mathbf{x}$ is a vector whose components are voltage and current differences from the considered operational fixed point, $\mathbf{b}$ is a vector of perturbations about the fixed point and $\mathbf{A}$ is the stability matrix.[1] When the perturbation is limited in time, $\mathbf{b}(t > t^*) = 0$, the formal solution for $t \gg t^*$ is $\mathbf{x}(t) = \exp[\mathbf{A}t]\mathbf{x}_0$. Therefore, if all eigenvalues of $\mathbf{A}$ are negative, the perturbed solution converges back to the initial operational fixed point and the system is judged linearly stable, i.e. stable under a small enough disturbance. The negative definiteness of $\mathbf{A}$ can be determined through Hurwitz stability criterion [17]. The trick is to arrange the coefficients of the characteristic polynomial of $\mathbf{A}$ into a Sylvester matrix $\mathbf{S}$. The roots of the polynomial – the eigenvalues of $\mathbf{A}$ – are then all negative if and only if all principal minors of $\mathbf{S}$ are positive.[2] Details of the procedure are given in Ref. [17]. The approach may be of interest for our problem, since adding few components requires to compute few more minors only.

The large signal case is of more importance to us, because, as we wrote above, any perturbation is potentially large in microgrids, where any load represents a non-negligible fraction of the total served load. Large disturbance stability is significantly harder to investigate than small-disturbance stability, because it requires not only to determine the existence of a basin of attraction surrounding the equilibrium point of the dynamical system under study, but additionally to characterize that basin, i.e. evaluate its spatial extension. Large disturbance stability is usually investigated via the Lyapunov method [22], which relies on constructing a function whose minimum lies at the equilibrium solution of the problem and whose gradient, giving the time-evolution of the system's degrees of freedom, is negative in a finite volume – the basin of attraction – around the equilibrium. Applications of the Lyapunov method to microgrids has been reviewed in Ref. [32].

[1] In our notation, matrices are represented as bold capital letters, while vectors are bold lowercase letters.
[2] The principle minors of a square matrix $\mathbf{S}$ are the determinants of all quadratic upper-left submatrices of $\mathbf{S}$.

The Lyapunov method is not applicable to our systems, however, precisely because of the presence of constant power loads. Refs. [19, 20] instead investigated the stability of low voltage DC microgrids using the Brayton-Moser method [33, 34]. The method represents the system dynamics in a quasi-gradient form $\mathbf{Q}\dot{\mathbf{x}} = -\nabla_x p + \mathbf{B}\mathbf{u}_s$. Then, if the matrix $\mathbf{Q}$ is positive definite in some coordinate and parameter region, there, the potential p is a non-increasing function. This allows one to certify large signal stability. The approach is reminiscent of the Hamiltonian method to classical physics [35]. For the case of RLC circuits, $\mathbf{Q}$ is a block-diagonal matrix, whose blocks are determined by the circuit inductance and capacitance matrices, $\mathbf{B}$ and $\mathbf{u}_s$ are the input matrix and voltages and the vector $\mathbf{x}$ of coordinates incorporates the current on the lines and the voltages on the nodes of the circuit. Some interesting applications of the Brayton-Moser method to few-bus circuits and microgrids are Refs. [36, 37, 38]. Using the so-called port Hamiltonian formalism may in particular be advantageous to incorporate electrical energy storage loads/sources into the analysis [39, 40]. Refs. [19, 20] used the Brayton-Moser method to investigate Ad Hoc microgrids, which are microgrids whose topology is not planned in advance – these are precisely the networks we are interested in, as they include, e.g. microgrids whose topology is constantly evolving, with components added as needs evolve. The discussion in the next section focuses on these works.

3.5 DC networks

The simple discussion in Section 3.2 illustrates how instabilities occur in DC microgrids with constant loads and gives hint about how such instabilities can be efficiently damped without jeopardizing the grid's energy-efficiency which, after all, is one of the main motivation for its deployment. It is however restricted to two-bus systems. We want to extend the analysis to more complicated microgrids, that furthermore may evolve with time as new components – loads or sources – are added to it. The most promising approach we found is the one constructed in Refs. [19, 20], which derives stability-aware design constraints for Ad Hoc DC microgrids guaranteeing transient stability of the microgrid. We give a succinct description of it here.

One considers that a microgrid is a networked collection of loads and voltage sources. The inner structure of sources and loads is shown in Fig. 5. The network is not specified except that it consist of n nodes labeled $i = 1, \ldots n$ (they are either sources or loads) connected to one another via some network of m resistive-inductive lines labeled by the indices of the nodes they connect, $\alpha = (i, j)$. The network is represented by its incidence matrix $\mathbf{A} \in \mathcal{R}^{m\times n}$ whose elements $A_{\alpha,i}$ give the admittance of the $\alpha^{\rm th}$ line, if one of it ends is the $i^{\rm th}$ node, and zero otherwise [41]. Voltages on all nodes in the microgrid are collected into a vector $\mathbf{v} \in \mathcal{R}^n$, as are currents, defined on the lines, $\mathbf{i} \in \mathcal{R}^m$. The state of the system is described by these two vectors. The action of the admittance matrix is then clarified by noting that $\mathbf{A}\mathbf{v}$ gives a vector of voltage differences across each line times the capacity of that line, while $\mathbf{A}^\top\mathbf{i}$ gives a vector of total currents flowing out of each node.[3]

Kirchhoff's laws of voltages and currents then read [neglecting the red stabilization cell in Fig. 5(b)]

$$C_k \partial_t v_k = -\frac{p_k}{v_k} - [\mathbf{A}^\top \mathbf{i}]_k, \tag{5a}$$

$$L_\alpha \partial_t i_\alpha = -\frac{R_\alpha}{i_\alpha} - [\mathbf{A}\mathbf{v}]_\alpha\,, \tag{5b}$$

where line inductances and resistances are L_α and R_α respectively, and each source and each load is characterized by a capacitance C_k. Sources are assumed to be perfect (they correspond to an inverter whose control is much faster than any of the time scales in the system), so that they inject at the rated voltage $v_k = E$. Voltages at loads fluctuate dynamically according to Eqs. (5).

The stability analysis of Eqs. (5) then proceeds along the lines of the Brayton-Moser theory. We do not describe it here but refer the interested reader to Refs. [19, 20] for further details. A worst-case scenario approach is followed, according to which a transient stability criterion is derived in the form of

[3] $\mathbf{A}^\top$ is the transpose of $\mathbf{A}$.

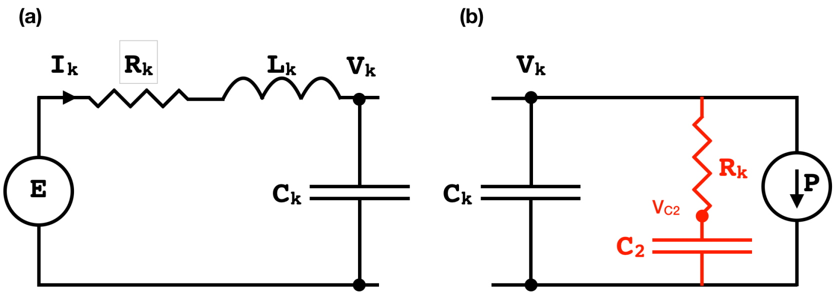

Figure 5: Inner structure of the elementary components of DC microgrids: (a) sources and (b) loads. They are connected to one another via some network (not shown, but discussed in the text). The stabilization cell shown in red in panel (b) may or may not be present.

a necessary condition to be satisfied by each load capacitance

$$C_k > \frac{\tau^{\max} p_k^{\max}}{V_{\min}^2}\,, \tag{6}$$

where $\tau^{\max} = \max_\alpha [L_\alpha / R_\alpha]$, $p_k^{\max}$ is the maximum power consumption at load k and $V_{\min}$ is a lower bound for the voltage across all loads.

It is important to search for necessary conditions for stability such as Eq. (6). Simultaneously, there is a need to go beyond that – Eq. (6) relies on several assumptions, on bounded loads and voltages for instances, which makes it more like a safe than a tight bound. The approach followed in Refs. [19, 20] seems quite promising, nevertheless, and one way to improve the results there could be to focus on questions pertaining to the addition of new components to an already stable grid.

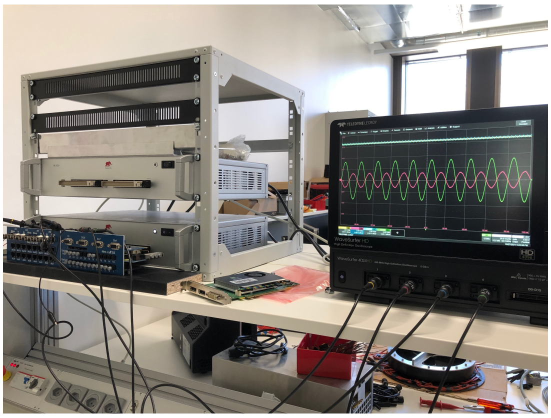

Figure 6: Real-time, hardware-in-the-loop simulation environment

3.6 Validation with virtual DC grids

The theoretical/numerical methods described above need to be tested before being implemented on a full-scale DC grid. To that end we foresee that hardware-in-the-loop approaches will prove instrumental, since they enable real-time studies of DC grids dynamic properties, from simple to more complex topology. Example for such a digital real-time platform is provided in Figure 6. In this paragraph we illustrate via few preliminary obtained results why we think such intermediate implementations between theoretical/numerical investigations and real-scale deployment is a very efficient way to validate theoretical results and guidelines.

Such tools do not allow the identification of necessary conditions for stability, at converter level or at global system level. However, they are very precious in that they enable the virtual experimental validation of all criteria and conditions to match for offering a guarantee of stability before real-scale implementation. An example of a circuit that can be implemented in a hardware-in-the-loop emulation is proposed in Figure 7.

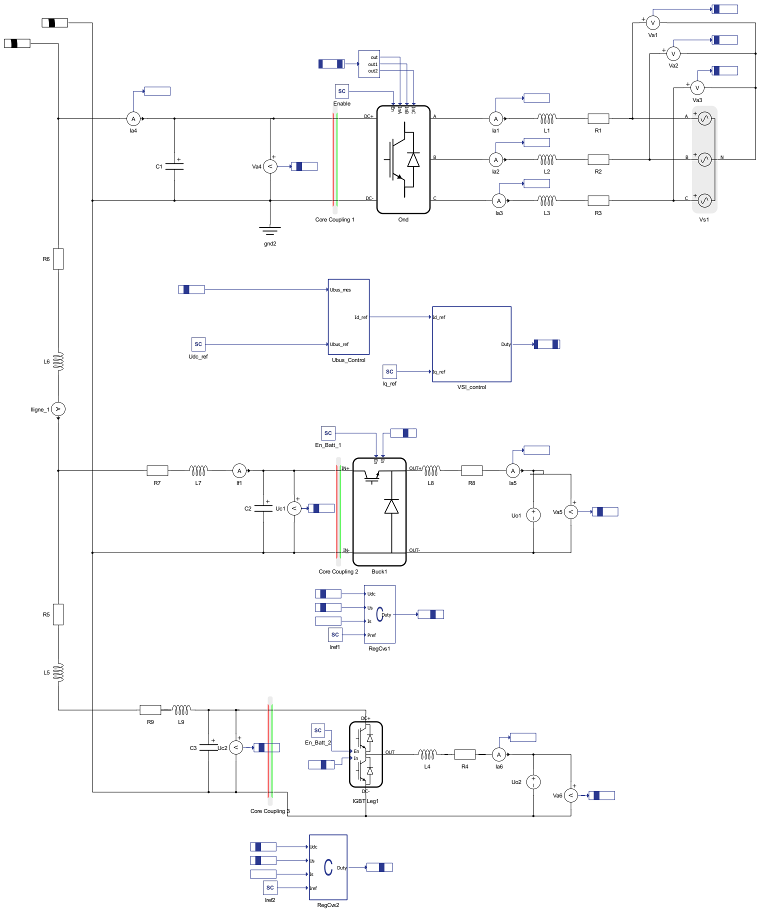

Figure 7: Model of an elementary DC bus

We use this model to illustrate how hardware-in-the-loop investigations on virtual DC grids may strengthen and validate purely theoretical guidelines derived from numerical investigations as sketched above. The model is composed of three sub-systems. From top to bottom: (i) a voltage source inverter to control a DC bus voltage through active power exchange with a three-phase low voltage grid, (ii) a non-reversible DC/DC converter feeding a DC voltage source, and (iii) a reversible DC/DC converter to charge/discharge batteries. The DC/DC converters are connected through LC input filters to a DC bus whose line impedances are taken into account. Models for each converter are harmonic models, where commutations on power switches are taken into account ($20kHz$ switching frequency). Each converter is controlled by its dedicated control algorithm, implemented in C++ as in conventional digital controllers.

To illustrate the power of the approach we first show in Fig. 8 how voltage stability in this DC bus system depends on grid topology. Figure 8(a) shows a first analysis, where only the reversible DC/DC converter is active and absorbs an increasing power because the battery it is connected to is charging. Once the power exceeds ca. $7.87kW$ an instability emerges. This agrees with usual considerations: instabilities occur above a given power level. One then enables the non-rreversible DC/DC converter. A similar analysis is shown in Figure 8(b). The non-reversible converter absorbs a $4kW$ constant power, which results in a voltage instability occuring at a much smaller power of ca. $5.4kW$.

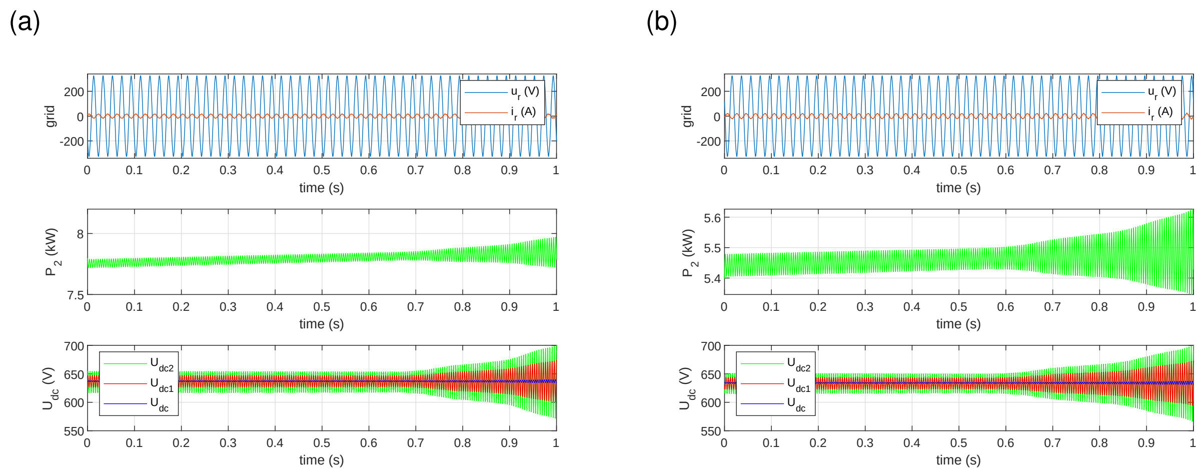

Figure 8: Operating point stability versus grid topology - (a) converter 1 is disabled - (b) converter 1 absorbs 4kW

Next, the occurence of instabilities under changes of the operating setpoint in the form of a stepwise increase of power through the non-reversible DC/DC converter is shown in Figure 9. The power through the non-reversible DC/DC converter increases from $6kW$ to $7.5kW$. In the first case, Figure 9(a), the DC bus is imperfectly damped, however it remains stable, with nonincreasing voltage oscillations after the power increase. In the second case, Figure 9(b), the non-reversible DC/DC converter is activated, absorbing a $6kW$ constant power on the DC bus. In this case, the sharp power increase leads to a voltage instability.

In all cases, the voltage source inverter is in charge to set the DC bus voltage to $650V$. These two examples nicely illustrate that local stability of one converter is affected by the grid topology and the operating setpoints of all components connected to the DC grid.

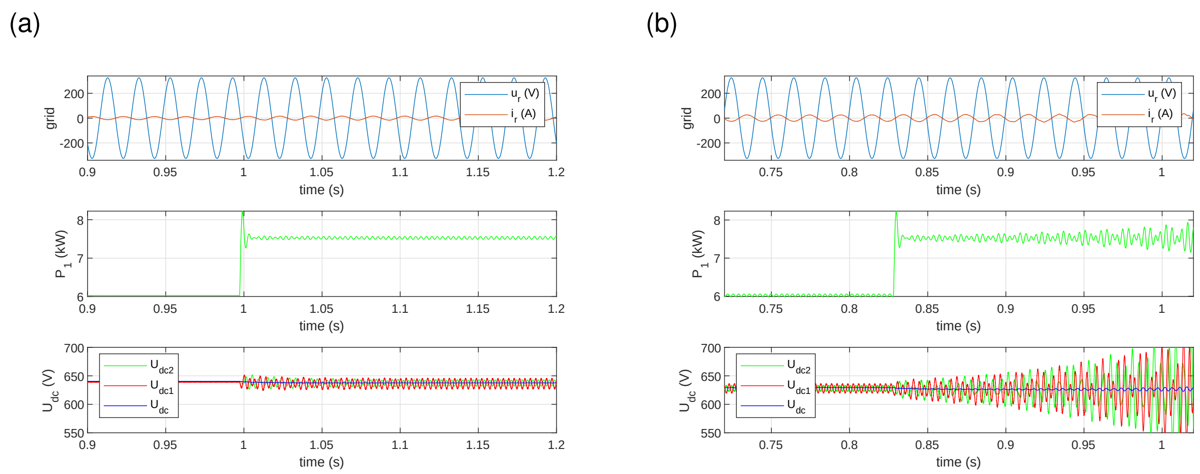

Figure 9: Sensitivity to power steps - (a) converter 2 is disabled - (b) converter 2 absorbs $6kW$

In complex systems such as the DC microgrids of interest here, there is often (always ?) significant, quantitative and qualitative differences between theoretical predictions and implemented, real-scale behaviors. Real-time simulations on hardware-in-the-loop virtual DC grids is an efficient way to circumvent this difficulty, test the validity of guidelines and design rules obtained from theoretical approaches discussed in section 3.5 before their implementations in real systems.

3.7 Main challenges

Having described the general stability problem of DC microgrids connected to AC grids, and proposed a general direction towards solving it, we list here what we think are the main questions and issues to be addressed in priority, to ensure the smooth deployment of DC microgrids.

First, we are looking to issue practical, general guidelines for guaranteeing the dynamical stability of *evolving* DC microgrids, as new components – electrical energy storage, renewable productions, loads etc. – are added and/or the topology of the DC grid itself is evolving – for instance because one goes from a single DC bus to a star network with radial ramifications. The guidelines need to be practical, i.e. implementable, limited in size and investment and guaranteeing to improve stability while not generating new issues on their own. Specifically, bounds like the one given in Refs. [19, 20] (Eq. (6) above) must be improved, for instance via the introduction of RC stabilizaton cells as in Fig. 5(b) [5]. Optimally, one would like to lift some assumptions on minimal voltages, maximal loads and so forth.

Second, stability needs to be investigated both from the point of view of the DC microgrid itself, but also from the point of view of the AC distribution grid to which it is connected.

Third, stabilization methods may rely on active – control based no power electronics, discussed in Section 3.3 – or passive – resistances and capacitors/filters, discussed in Section 3.2 – components. Both solutions should be investigated and their pros and cons evaluated and compared.

Fourth, special attention should be given to effective coupling arising between components, in particular converters and active components. Such coupling may eventually lead to cooperative modes where components synchronize across the DC grid and oscillate coherently, which could be detrimental to their stability. Efficient ways to suppress this coupling and to prevent components from talking to one another should be found.

Fifth, it is desirable to investigate whether stabilization is more efficiently achieved by adding passive stabilization units at each components/in parallel to each converter, or at few, well chosen places. Of key importance is to be able to guarantee stabilization at limited financial cost – the very reason why microgrid are of interest is that they reduce costs for integrating inverter-connected components into the electric grid, and it should remain so.

From a methodology point of view, we think that a three-pronged approach would be efficient where

• initial stability investigations proceed along theoretical/numerical lines, including small-signal stability analysis based on system linearization and large disturbance analysis based on Lyapunov or Brayton-Moser methods,

• the ensuing guidelines are tentatively implemented on hardware-in-the-loop virtual DC grids in order to validate them, and

• a large-scale implementation is finally done on a real, large-scale demonstrator such as the gridlab at the School of Engineering at HES-SO/Valais.

4 References

[1] *World energy outlook 2017*, International Energy Agency (2017).

[2] W. Inam, *Adaptable Power Conversion for Grid and Microgrid Applications*, PhD Thesis, MIT (2016).

[3] J.J. Justo, F. Mwasilu, J. Lee, and J.W. Jung, *AC-microgrids versus DC-microgrids with distributed energy resources: A review*, Renewable and Sustainable Energy Reviews **24**, 387–405 (2013).

[4] N.O. Sokal, *System oscillations caused by negative input resistance at the power input port of a switching mode regulator, amplifier, dc/dc converter, or dc/ac inverter*, 1973 IEEE Power Electronics Specialists Conference, 138 (1973).

[5] P. Barrade, H. Piquet, and Y. Chéron, *Méthode d'étude de la stabilité des ensembles convertisseurs-filtres*, J. Phys. III France **6**, 91–104 (1996).

[6] X.G. Feng, J.J. Liu, F.C. Lee, *Impedance specifications for stable DC distributed power systems*, IEEE Transactions on Power Electronics **17**, 157 (2002).

[7] S. Girinon, H. Piquet, N. Roux and B. Sareni, *Analytical input filter design in DC distributed power systems approach taking stability and quality criteria into account*, 13th European Conference on Power Electronics and Applications, 1 (2009).

[8] A. Kwasinski, C. N. Onwuchekwa, *Dynamic behavior and stabilization of DC microgrids with instantaneous constant-power loads*, IEEE Transactions on Power Electronics **26**, 822 (2011).

[9] A. Riccobono and E. Santi, *Comprehensive review of stability criteria for DC power distribution systems*, IEEE Trans. Ind. Appl. **50**, 3525 (2014).

[10] S. Anand and B. G. Fernandes, *Reduced-order model and stability analysis of low-voltage DC microgrid*, IEEE Trans. Ind. Electron. **60**, 5040 (2013).

[11] S. Sanchez, R. Ortega, G. Bergna, M. Molinas, and R. Grino, *Conditions for existence of equilibrium points of systems with constant power loads*, 52nd Annual Conference on Decision and Control (CDC), 3641–3646 (2013).

[12] S. Bolognani and S. Zampieri, *On the Existence and Linear Approximation of the Power Flow Solution in Power Distribution Networks*, IEEE Transactions on Power Systems **99**, 1–10 (2015).

[13] G. Cezar, R. Rajagopal, and B. Zhang, *Stability of interconnected DC converters*, Proc. 54th IEEE Conf. Decis. Control (CDC) (2015).

[14] J. W. Simpson-Porco, F. Dörfler, and F. Bullo, *On Resistive Networks of Constant-Power Devices*, IEEE Transactions on Circuits and Systems II: Express Briefs **62**, 811–815 (2015).

[15] N. Barabanov, R. Ortega, R. Griño and B. Polyak, *On existence and stability of equilibria of linear time-invariant systems with constant power loads*, IEEE Trans. Circuits Syst. I **63**, 114 (2016).

[16] M. Farrokhabadi et al. (IEEE PES Task Force on Microgrid Stability Definitions, Analysis, and Modeling), "Microgrid Stability Definitions, Analysis, and Examples", IEEE Trans. Power Syst. 35, 13-29 (2020).

[17] K.J. Astrom and R. Murray, *Feedback Systems: An Introduction for Scientists and Engineers*, Princeton University Press (2010).

[18] J.A. Belk, W. Inam, D.J. Perreault, and K. Turitsyn, *Stability and control of ad hoc dc microgrids*, 55th Conference on Decision and Control (CDC), 3271–3278 (2016).

[19] K.A. Cavanagh, *Stability-constrained design for low voltage DC microgrids*, PhD Thesis, MIT (2018).

[20] K. Cavanagh, J.A. Belk, and K. Turitsyn, *Transient Stability Guarantees for Ad Hoc DC Microgrids*, IEEE Control Systems Letters **2**, 139 (2018).

[21] P. Magne, B. Nahid-Mobarakeh, S. Pierfederici, *General active global stabilization of multiloads DC-Power networks*, IEEE Transactions on Power Electronics **27**, 1788 (2012).

[22] J.P. Hespanha, *Linear Systems Theory*, Princeton University Press (2018).

[23] N. Bottrell, M. Prodanovic, T. C. Green, *Dynamic stability of a microgrid with an active load*, IEEE Transactions on Power Electronics **28**, 5107 (2013).

[24] R.H. Park, *Two reaction theory of synchronous machines*, AIEE Transactions **48**, 716 (1929).

[25] W.C. Duesterhoeft, M.W. Schulz, and E. Clarke, *Determination of instantaneous currents and voltages by means of alpha, beta, and zero components*, Transactions of the American Institute of Electrical Engineers **70**, 1248 (1951).

[26] IEEE Standard for the Specification of Microgrid Controllers, IEEE Standard 2030.7-2017 (2017).

[27] Y. Han, H. Li, P. Shen, E.A.A. Coelho, and J.M. Guerrero, *Review of Active and Reactive Power Sharing Strategies in Hierarchical Controlled Microgrids*, IEEE Trans. Power Electron. 32, 2427-2451 (2017).

[28] H. Bevrani, B. François, and T. Ise, *Microgrid Dynamics and Control*, Wiley, Hoboken-USA (2017).

[29] C. De Persis, E.R.A. Weitenberg, and F. Dörfler, *A power consensus algorithm for dc microgrids*, Automatica **89**, 364–375 (2018).

[30] F. Gao, R. Kang, J. Cao, and T. Yang, *Primary and Secondary Control in DC Microgrids: A Review*, J. Modern Power Syst. Clean Energy 7, 227-242 (2019).

[31] Y. Khayat, M. Naderi, Q. Shafiee, Y. Batmani, M. Fathi, J.M. Guerrero, and H. Bevrani,, *Decentralized Optimal Frequency Control in Autonomous Microgrids*, IEEE Trans. Power Syst. 34, 2345-2353 (2019).

[32] M. Kabalan, P. Singh, and D. Niebur, *Large signal lyapunov-based stability studies in microgrids: A review*, IEEE Transactions on Smart Grid **8**, 2287–2295 (2017).

[33] R. Brayton and J. Moser, *Some Results on the Stability of Nonlinear Networks Containing Negative Resistances*, IEEE Transactions on Circuit Theory **11**, 165–167 (1964).

[34] R. Brayton and J. Moser, *A theory of nonlinear networks. I and II*, Quarterly of Applied Mathematics **22**, 1 (1964); **22**, 81 (1964).

[35] D. Jeltsema and J. Scherpen, *Multidomain modeling of nonlinear networks and systems*, IEEE Control Systems Magazine **29**, 28–59 (2009).

[36] M. Belkhayat, R. Cooley, and A. Witulski, *Large signal stability criteria for distributed systems with constant power loads*, in Proc. IEEE Power Electron. Spec. Conf, 1333 (1995).

[37] A. Griffo, J. Wang, and D. Howe, *Large signal stability analysis of dc power systems with constant power loads*, in Proc. Vehicle Power Propulsion Conf., 1 (2008).

[38] K.C. Kosaraju, M. Cucuzzella, J.M.A. Scherpen, and R. Pasumarthy, *Differentiation and passivity for control of Brayton-Moser systems*, IEEE Transactions on Automatic Control **66**, 1087 (2021).

[39] D.I. Makrigiorgou and A.T. Alexandridis, *Stability Analysis of DC Distribution Systems with Droop-Based Charge Sharing on Energy Storage Devices*, Energies **10**, 433 (2017).

[40] B. Severino, et al., *Enhancing Transient Stability of DC Microgrid by Enlarging the Region of Attraction Through Nonlinear Polynomial Droop Control*, IEEE Transactions on Circuits and Systems **66**, 4388 (2019).

[41] N. Biggs, *Algebraic graph theory*, 2nd ed., Cambridge University Press (1993).