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APPLICATIONS OF MAGNETIC »POWER PRODUCTION« AND ITS ASSESSMENT

CALCULATION METHODS

Final report: Appendix 3

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Impressum

Datum: *5. Juni 2008*

Im Auftrag des Bundesamt für Energie
Forschungsprogramm Elektrizitätstechnologien und -anwendungen

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BFE-Vertrags- und Projektnummer: *101776 / 152190*

Bezugsort der Publikation: www.energieforschung.ch / www.electricity-research.ch

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1. INTRODUCTION

An important information for a design or analysis of a magnetic energy conversion machine, also named power generator, is the specific work capacity of the magnetocaloric material. The magnetocaloric materials are limited by the temperature span, in which they can operate. From an engineering point of view, this is directly related to the total entropy and the adiabatic temperature change of the material. Both depend on the magnetic field variation. Because the magnetocaloric materials show the largest potential near their Curie temperature, it is obvious that large temperature spans between the heat source and the heat sink require layered magnetocaloric material beds, each of them adapted to a certain temperature interval. A large number of different magnetocaloric materials may be found in references [1-4]. Among all the magnetocaloric materials, Gadolinium may be used as a reference material, similar as the ideal gas in conventional refrigeration. The reason is that its properties are well defined by the application of the mean field theory [5] and there is no need of experimental data. Based on such data, it is possible to define the maximal specific work of the magnetocaloric material.

1.1 THE MAXIMAL SPECIFIC WORK

Each magnetocaloric material shows a maximal specific work (for a certain temperature difference between the heat source and the heat sink) in the vicinity of its Curie temperature. There, the adiabatic temperature change as well as the entropy change of the magnetocaloric material reach their maxima. In magnetocaloric power conversion, it therefore does not make sense to operate with a single magnetocaloric material, because its specific work capability is rapidly decreasing with an increased temperature span around the Curie temperature (see differences in distances for the two isofield lines in Figure 1).

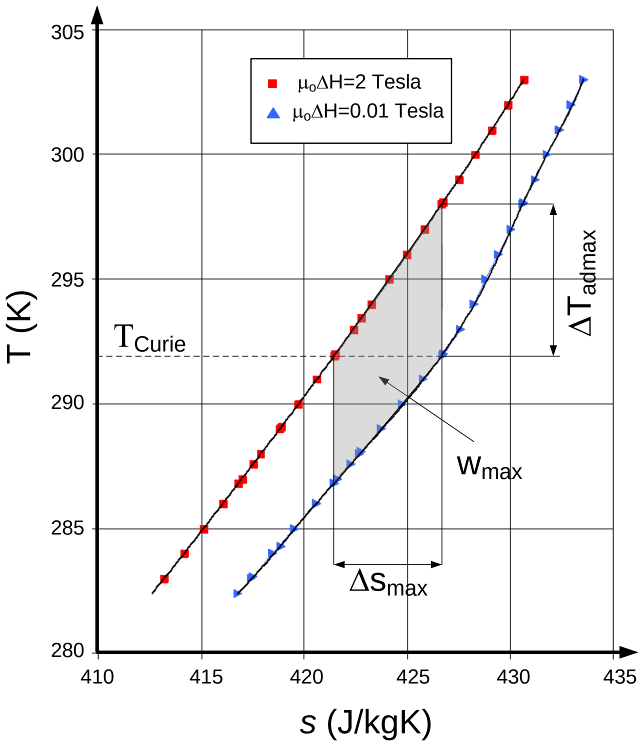

Figure 1: An example of the maximal specific work obtained for Gadolinium in the limits of the adiabatic temperature change, determined by the magnetic field change in the vicinity of the Curie temperature.

In order to obtain the maximal possible work for a given (large) temperature span between the heat source and the heat sink, several layered (tuned) magnetocaloric materials lead to much better characteristics than a single material. However, this also requires a regenerative cycle or a cascade system, where each of the layered materials undergoes its own thermodynamic cycle performed at conditions close to its Curie temperature. The temperature span, where a single material can efficiently produce work, is therefore limited by the given magnetic field change. For the purpose of the analysis, Gadolinium properties are taken into consideration [5]. The magnetocaloric material is assumed to undergo a Brayton power thermodynamic cycle, as presented in Figure 1. Additionally, the assumption is made that all the magnetocaloric materials – layered to perform a multi-stage or a cascade process – have the same properties as Gd, but shifted to different Curie temperatures.

The maximal specific work for a single magnetocaloric material is defined as follows:

$$w_{(\max)} = \int_{s(T_C,\,\mu_0 H_0)}^{s(T_C+\Delta T_{ad}(T_C),\,\mu_0 H_0)} T\,ds \;-\; \int_{s(T_C-\Delta T_{ad}(T_C),\,0)}^{s(T_C,\,0)} T\,ds \tag{1}$$

In the analysis, the magnetic flux density is assumed to vary from 0 T to $\mu_0 H_0$= 1, 1.5, 2, 2.5, 3, 5, 10 and 20 T (T=Tesla). Because the time variation of magnetization/demagnetization for a magnetocaloric material in a real system determines the frequency of operation, it is possible to define the theoretical maximal specific power as follows:

$$P_{(\max)} = w_{(\max)} \cdot f \tag{2}$$

Figure 2 shows the specific power according to Eq. (2) calculated for Gadolinium. One should notice that even for a single magnetocaloric material high values are obtained.

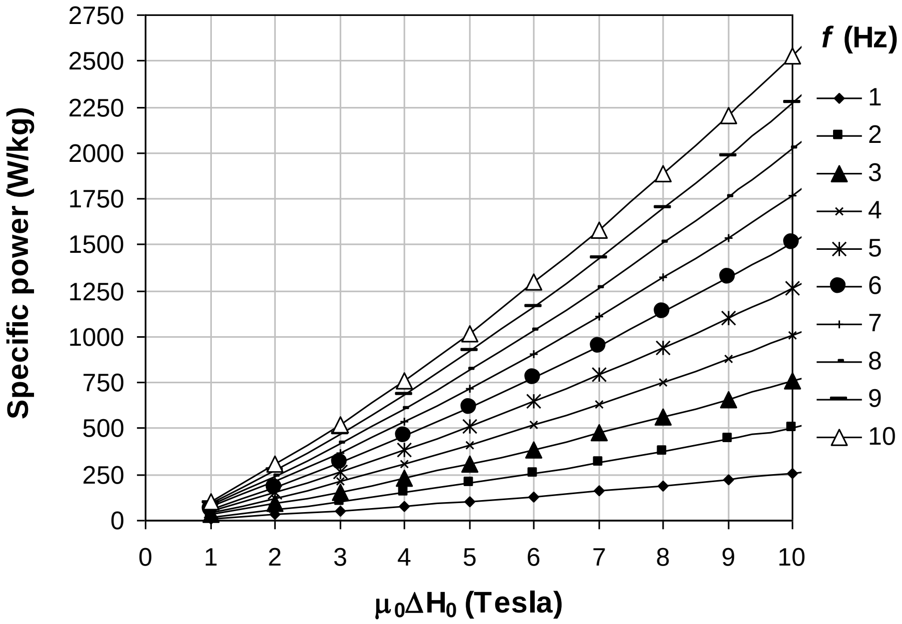

Figure 2: Specific power of Gadolinium depending on different magnetic field changes and different frequencies of operation.

In the analysis it was assumed that each layered magnetocaloric material is able to deliver a specific power just as presented in Figure 2. This also means that each layered material operates only around its Curie temperature with its defined adiabatic temperature change in order to perform e.g. a Brayton power generation cycle. For a multistage (regenerative) machine, the maximal specific work comprises contributions of each layer of different magnetocaloric material. The ideal maximal specific work output of a multi-stage machine is then:

$$w_{(\max multi)} = w_{(\max)} \frac{T_H - T_C}{\Delta T_{ad}} \tag{3}$$

where it is assumed that each layered material operates only within the temperature interval equal to the adiabatic temperature change. The corresponding ideal maximal electric power of such a machine is:

$$P_{(\max multi)} = w_{(\max multi)} \cdot f = P_{ideal} \quad (4a,b)$$

Equations 3 and 4 imply that the number of magnetocaloric materials required being able to cover the whole temperature span between the temperatures of the heat source and the heat sink is equal to the ratio of the whole temperature span and the adiabatic temperature difference, which occurs at the specific magnetic field change.

Note that this number is not the same as the number of stages, because the first is related to the ideal output power from a magnetocaloric power machine and the latter is increased by taking into account heat loss irreversibility's.

1.2 THE GEOMETRY

The geometrical characterization and the related equations have already been presented in references [6,7].

2. CHARACTERISTICS OF A MAGNETIC POWER GENERATOR

2.1. THERMODYNAMIC AND EXERGY EFFICIENCY

The thermodynamic efficiency of a heat engine is the ratio between the mechanical or even electrical output power and the heat power added to the system:

$$\eta_{th} = \frac{P_{el}}{\dot{Q}_H} \quad (5)$$

In the ideal (Carnot cycle) power generator, the thermodynamic efficiency is defined by the temperature levels of the heat source and the heat sink:

$$\eta_{Carnot} = \frac{T_H - T_C}{T_H} = \frac{P_{ideal}}{\dot{Q}_H} \quad (6)$$

The exergy efficiency of the ideal Carnot cycle is one, because this cycle is fully reversible. It means that the whole exergy input (e.g. exergy of heat) is converted into useful work. Since the electric power is obtained by an electric generator with a certain efficiency, the mechanical power to drive it needs to be larger. This can be taken into account by introducing an efficiency of the generator, which in our analysis is estimated to be 90% (97% in the case of a superconducting power generator):

$$\eta_{Mot} = 0.9 \quad (0.97) \quad (7)$$

The mechanical work obtained from a rotation of magnetocaloric material is partly dissipated by eddy currents and by a hysteresis effect of the magnetocaloric material. To describe these effects, two additional efficiencies were introduced:

$$\eta_{Hyst} = 0.97 \qquad \eta_{Eddy} = 0.95 \quad (8a,b)$$

The temperature difference between the heat source and the heat sink is the maximal temperature difference available for power production:

$$\Delta T_{tot} = T_H - T_C \tag{9}$$

This temperature difference between a heat source and a heat sink is partitioned into multiple stages (or regenerative cycles). However, there exists also a “temperature loss” due to the irreversible heat transfer between the magnetocaloric material and the working fluid. Other “temperature losses” occur due to irreversible heat transfers in the external cold and hot heat exchangers. Contrary to the mechanisms in refrigerators or heat pumps, ΔT_{tot} is not a temperature difference which is "built up" by a certain number of stages and the occurring adiabatic temperature changes. In this case it presents the maximal possible temperature span. In a magnetic power generator, as consequence of the present heat transfer processes, different “temperature losses” occur. These may be listed as: one at the source (ΔT_{source}), one at the sink (ΔT_{sink}) and one between the material and the fluid on each side of each stage ($\Delta T_{stage,\ cold}$ and $\Delta T_{stage,\ hot}$). In a first approximation we assume that the temperature differences at the source and the sink are identical. This is indeed the case as a design will usually involve symmetric conditions. It may also be assumed that in all stages the same fluid will be applied:

$$\Delta T_{stage} = \Delta T_{stage,cold} = \Delta T_{stage,hot} \tag{S10}$$

and

$$\Delta T_{loss} = \Delta T_{stage,cold} + \Delta T_{stage,hot} = 2\ \Delta T_{stage} \tag{S11}$$

In Equations (S10) and (S11) S denotes a special case. The total temperature difference between the heat source and the heat sink is expressed as follows:

$$\Delta T_{tot} = n \cdot \Delta T_{ad} + \Delta T_{source} + \Delta T_{\sin k} \tag{12}$$

where *n* is the number of stages. This is calculated by the given temperature differences in Eq. (12). It means that the temperature difference, which is available for the magnetocaloric power generator, is reduced by the temperature losses at the two external heat exchangers for the heat sink and the heat source. The real temperature difference, which is available to perform work, is further reduced in the magnetocaloric material, because of the internal heat transfer losses (e.g. between stages of the magnetocaloric material):

$$\Delta T_{ext} = n \cdot \left(\Delta T_{ad} - \Delta T_{stage,cold} - \Delta T_{stage,hot}\right) \tag{13}$$

From equation (12) one may immediately conclude that if the internal irreversible heat transfer losses between the stages overcome the adiabatic temperature change, no work will be performed. It also follows by Equation (S11):

$$\Delta T_{ext} = n \cdot \left(\Delta T_{ad} - 2\,\Delta T_{stage}\right) \tag{S14}$$

The efficiency of a multistage machine is then defined to be:

$$\eta_{thmulti} = \frac{\Delta T_{ext}}{T_H} \tag{15}$$

The exergy efficiency of the heat transfer processes is described as the ratio between the terms shown in Equation (15) and (6):

$$\eta_{Multi} = \frac{\eta_{thmulti}}{\eta_{Carnot}} \qquad (16)$$

Applying Eq.'s (6),(14) and (16), the exergy efficiency of the heat transfer processes is defined to be:

$$\eta_{Multi} = \frac{n\left(\Delta T_{ad} - 2\Delta T_{stage}\right)}{T_H - T_C} = \frac{n\left(\Delta T_{ad} - 2\Delta T_{stage}\right)}{n\ \Delta T_{ad} + \Delta T_{source} + \Delta T_{\sin k}} \qquad (S17a,b)$$

Furthermore, we define that the heat losses contribute to the total basic exergy potential by some amount:

$$\eta_{Gen} = 0.95 \qquad (18)$$

We assume that the pressure loss is the same in all stages. Then, if the loss by pressure drop of one stage is P_{hyd}, for n stages it follows:

$$P_{Hyd} = n\ P_{hyd} \qquad (19)$$

Note that the different size of initial letters denotes the "specific power loss" of a single stage (P_{hyd}) and that of the total machine (P_{Hyd}). Now it follows that:

$$\eta_{Hyd}\ \eta_{Carnot} = \eta_{Hyd}\ \frac{P_{ideal}}{\dot{Q}_H} = \frac{P_{ideal} - P_{Hyd}}{\dot{Q}_H} \qquad (20)$$

Then one has:

$$\eta_{Hyd} = \frac{1}{\eta_{Carnot}}\ \frac{P_{ideal} - P_{Hyd}}{\dot{Q}_H} \qquad (21)$$

respectively:

$$\eta_{Hyd} = 1 - \frac{P_{Hyd}}{P_{ideal}} \qquad (22)$$

Now the final efficiency of a machine with several stages is given:

$$\eta_{th} = \xi\ \eta_{Carnot} \qquad (23)$$

with:

$$\xi = \eta_{Mot}\, \eta_{Eddy}\, \eta_{Hyst}\, \eta_{Multi}\, \eta_{Gen}\, \eta_{Hyd} \tag{24}$$

For the case of superconducting power generation 10% more losses may be taken into account due to the high temperature superconducting cooling system and the electric energy for magnetization. This leads to the thermodynamic efficiency:

$$\eta_{Final} = \eta_{Carnot} \cdot \eta_{Mot} \cdot \eta_{Eddy} \cdot \eta_{Hyst} \cdot \eta_{Multi} \cdot \eta_{Gen} \cdot \eta_{Hyd} \cdot \eta_{Sc} \tag{25}$$

where the last term in Eq. (25) presents an efficiency of 90%.

The theory which has been presented in this report is based on the assumption that the losses of heat transfer (*Multi*) and by pressure drop of the flow (*Hydr*) occur in a quasi-homogeneous manner distributed over the entire temperature range of the refrigerator. If different parts of a machine have different high losses, this theory will fail. Derivations for more complex situations with different sized stages are published in reference [8]. For applications with superconducting magnets, the efficiency is further reduced due to the power demand for the magnetic field source, as well as the cryogenic system. It contributes to a loss of approximately 10% efficiency in large scale applications.

3. EVALUATION OF THE TOTAL MASS AND VOLUME

The determination of the total mass and volume of a magnetic power generator starts with the electric output power of the device. Based on this and the exergy efficiency for certain operating conditions, the total mass of the magnetocaloric material is calculated by the following equation:

$$m_{mc} = \frac{P_{el}}{\xi \cdot P_{(\max multi)}} \tag{26}$$

Knowing the volume fraction ψ of the magnetocaloric porous structure, one is able to define the total volume that this structure occupies:

$$V_{ps} = \frac{m_{mc}}{\rho_{mc}\, \psi} \tag{27}$$

When taking into account that one half of the magnetocaloric ring is in the magnetic field and the other half outside of it, this defines the minimal gap volume:

$$V_{gap} = \frac{m_{mc}}{2\rho_{mc}\, \psi} \tag{28}$$

The space leading to this volume is surrounded by permanent or other kind of magnets.

Based on a numerical analysis, which was performed by a finite element method program, the dependence on the gap and the applied magnetic field is approximately defined by the following empirical relation:

$$V_{mag} = x V_{gap} = \left[(\mu_0\, \Delta H_0)^2 - \mu_0\, \Delta H_0 + 2 \right] V_{gap} \tag{29}$$

The factor x in Equation (29) is graphically given in Figure 3. The higher the magnetic field is, the higher x must be chosen.

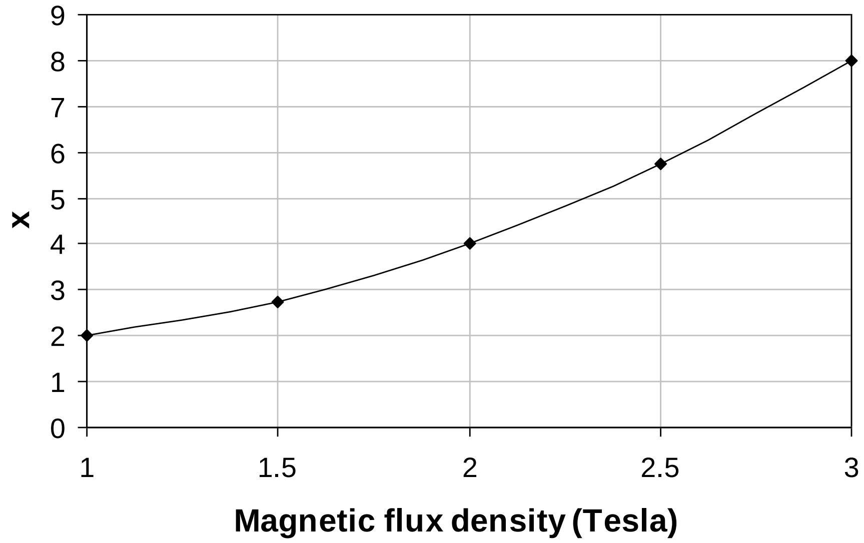

Figure 3: A multiplication factor for large scale refrigerators using permanent magnets.

Equation (29) was used for large scale applications, assuming a better compactness of the machines. For small scale applications an even stronger restriction must be made, because small volumes of magnetocaloric material require a rather large volume of the magnet assembly. This effect is taken into consideration by a size-dependent correction factor f:

P_{el}=10 kW	f=3
P_{el}=100 kW	f=2
P_{el}=1 MW	f=1

The magnet assembly mass was then defined using a density of 7500 kg/m^3. The other components such as pipes, valves, etc. also contribute to the total mass and volume. Based on the available data from the producers the mass and volume of the magnets assembly involving pipes, valves, etc. are estimated (for a minimal P_{el}=5 kW) by the following equations:

$$m_{ass} = P_{el} \cdot 14.14 \cdot 10^{-3} - 23.46 \quad (30)$$

$$V_{ass} = P_{el} \cdot 4.24 \cdot 10^{-6} + 0.0708 \quad (31)$$

In these empirical equations the units must be chosen in the following manner:

$[P_{el}]$ = W

$[m_{ass}]$ = kg

$[V_{ass}]$ = m^3

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5. SYMBOLS

Standard

f	frequency (Hz, s^{-1})
H	magnetic field strength (A m^{-1})
P	power (W)
Q	heat (J)
$\dot{q}$	specific heat power (W kg^{-1})
$\dot{Q}$	heat (W)
s	specific entropy (J $kg^{-1}K^{-1}$)
T	temperature (K, °C)
V	volume (m^3)
w	specific work (J/kg)
W	work (J, Nm)
m	mass (kg)

Greek

μ	magnetic permeability (Wb $A^{-1}m^{-1}$, NA^{-2}, H m^{-1})
ψ	volume fraction
η	efficiency
ξ	exergy efficiency
ρ	density (kg m^{-3})

Indices

ad	adiabatic
admax	adiabatic maximal
ass	assembly
C	Curie, cold, sink
Eddy	eddy currents
el	electric
ext	real temperature difference
Gen	general
Gap	air gap
H	hot source
hyd	hydraulic for one stage
Hyd	hydraulic for *n* stages
Hyst	hysteresis
mag	magnets
max	maximal
maxmulti	maximal multi stage
mc	magnetocaloric
Mot	motor
Multi	multi stage
n	number of stages
ps	porous structure
Sc	superconducting
tot	total, overall
th	thermodynamic
thmulti	thermodynamic multi stage
0	external, vacuum