Evaluating the methodology for the individual risk assessment for passengers in railway transport

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Management summary The Swiss Federal Office of Transport (FOT) developed a method for evaluating the acceptance of the individual risks to passengers and railway employees (see Slovak et al., 2012; Baeriswyl et al., 2013). Our aim was to assess the current method applied by FOT, to validate assumptions of the current approach and to suggest improvements of the current approach if needed. We focused on the individual risk for passengers.

It is important for railway companies to assess individual risks of a particular technological system or a planned structure as well as the overall individual risks induced by the railway transport. However, investigating an occurrence of a rare event (e.g. death of a passenger in a train collision) leads to estimating extremely small probabilities which is usually imprecise. To solve this problem, we proposed a new approach of individual risk assessment. Our approach is based on viewing the individual risk as a random variable and calculating the probability, that the individual risk is acceptable. The proposed method is demonstrated on actual data collected by FOT.

Recommended changes in the individual risk assessment methodology

- Comparison with the acceptance criterion derived from the minimum endogenous mortality is possible only in concern of fatalities. Therefore, the value of the collective risk for passengers should be adapted.
- The estimation of the average travel distance should be related to the whole railway network of Switzerland and not to the largest Swiss railway company SBB only.
- Statistical confidence of the estimated individual risks has to be considered. We propose the application of the Bayes' theorem providing additional information for more confidential decision-making. This extended approach enables more realistic setting of several parameters that were originally set very conservatively.
- We recommend a more transparent calculation of the passengers individual risk. By the use of the proposed pre-generated tables of acceptance for evaluation of the statistical confidence, there is no additional effort for a user of the risk assessment methodology.

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Table 1: List of abbreviations.

Abbreviation	**Meaning**	**FOT equivalent** (Baeriswyl et al., 2013)	**FOT equivalent** (Slovak et al., 2012)
R_{iP}	Overall individual risk for a passenger	r_{iE}	R_{iEP}
R_{iAccP}	Threshold for the overall individual risk of passengers	$R_{iEakzFg}$	R_{iEaccP}
$R_{i\phi P}$	Estimated average basic individual risk to passengers in CH	R_{iEB}	$R_{iE\phi P}$
$R_{c\phi}$	Estimated average basic collective risk to passengers in CH	R_k	R_c
E_{iMaxP}	Maximal period of exposure of an individual passenger	E_{maxFg}	E_{maxP}
E_P	Cumulative period of exposure of all passengers	E_P	E_P
R_{iCase}	Estimated case-specific individual risk	R_{iEFall}	R_{iECase}
R_{cCase}	Estimated case-specific collective risk	R_{KFall}	R_{cCase}
n_{iCase}	Number of individual passenger's case-specific expositions	W_{max}	W_{max}
N_{PCase}	Number of case-specific expositions of all passengers	N_{FgFall}	N_{PCase}
W_{Case}	Predicted frequency of repetition of the case-specific risks during a passengers average journey	W_{Fall}	W_{case}
N_{OC}	The expected maximum occurrence of risks arising from other cases within an average journey	$N_{Fälle}$	N_{ZR}
not needed	Average duration of a rail journey	$E_{\phi Fg}$	$E_{\phi P}$
not needed	Maximum permitted individual risk based on the period of exposure	$R_{iEmaxFg}$	R_{iEmaxR}
not needed	Permitted individual risk to a person encountering a case-specific risk during a single occasion	$R_{i1akzFg}$	$R_{iNExposureAcc}$

Introduction

The Swiss Federal Office of Transport (FOT) developed a method for evaluating the acceptance of the individual risks to passengers and railway employees (see Slovak et al., 2012). Our aim was to assess the current method applied by FOT, to validate assumptions of the current approach and to suggest improvements of the current approach if needed. We focused on the individual risk for passengers.

The individual risk for passengers is investigated in the first section section of the evaluation report. First, the threshold for the overall individual risk is defined. Afterwards, the original approach of the FOT is discussed. The improvement of the original approach is described in sections 1.3 and 1.5. The use of both the original approach and the proposed approach is illustrated on actual examples throughout the first section. Practical use of the proposed approach is described in Section 2. Finally, recommendations and further remarks are given in the last section. The statistical details of the proposed approach and the theoretical justification of the statements and conclusions are described in Appendix.

1 Individual risk

The risk assessment involves proving that the individual risk, which is a risk for an individual, is acceptable. The acceptance level of the individual risk was defined as $1/20$ of the minimum endogenous mortality (MEM). MEM can be taken from the minimum human mortality excluding deaths caused by illnesses, accidents, violence, immaturity and congenital deformity (see Kuhlmann, 1981). Nowadays, MEM accounts for $2 \cdot 10^{-4}$ F/Y (see Slovak et al., 2012). Therefore, the European railway standard EN 50126 (1999) recommends to set the threshold of the individual risk

$$R_{iAccP} = 1 \cdot 10^{-5} \text{ F/Y}$$

as a non-significant increase in the total risk for a human ($1/20$ of MEM).

It is reasonable to split the overall individual risk into two types: basic risk and case-specific risk. The basic risk can be estimated from the empirical data. It expresses the continuous exposure of an individual to the risk and reflects the current state of the railway network. The case-specific risk can be derived from estimated operation parameters and collective risk caused by realization of a particular project.

First, we have to prove that the basic risk is lower than R_{iAccP}. Regarding a particular project, the next step is the analysis of the case-specific risk. We may accept the case-specific risk only if we are able to show that the case-specific risk is together with the basic risk still below R_{iAccP}.

1.1 Basic risk – original approach

The basic collective risk for passengers in Switzerland was estimated from the actual data collected by FOT over the period 2000 – 2014. Four fatalities occurred during the studied period due to accidents in full responsibility of the railway companies (like collisions, derailments). FOT suggested including a margin accounting for other two fatalities (representing a potential accident with 6 fatalities every 30 years) which leads to the basic collective risk $R_{c\phi} = 0.4$ F/Y.

The method of FOT currently uses the formula from Slovak et al. (2012), page 3, to calculate the point estimate of the basic individual risk:

$$R_{i\phi P} = (R_{c\phi} E_{iMaxP})/E_P \quad [\mathrm{F/Y}], \tag{1}$$

where $R_{i\phi P}$ is the estimated individual risk (point estimate), $R_{c\phi}$ stands for the collective risk, E_{iMaxP} is the time of exposure of an individual and E_P is the cumulative period of exposure of all people in Switzerland.

The other two formulas described in Slovak et al. (2012), pages 3 and 4, are less complex and flexible than relation (1). The first of them takes into account only the maximum number of occasions during which the system is used by an individual and can be viewed as a special case of (1) after transforming the maximum number of occasions into the maximum exposure time of an individual. The second one substitutes one year for E_{iMaxP} in (1). Hence, it is also a special case of (1) and reasoning depicted in Figure 3 in Slovak et al. (2012), page 6, is necessary. Thus, the use of this relation is more complicated. In addition, the units in this relation seems to be incorrect. Therefore, we use formula (1) for computing point estimates of individual risks.

According to the statistical data provided by FOT, a person living in Switzerland has an exposition time of 6.4 min per day in average (Federal Office of Statistics 2010). Considering the Swiss population from 2010, it leads to $E_P = 6.4 \cdot 365 \cdot 7870134$ min ≈ 34978.4 Y. Hence, the estimated individual risk is below R_{iAccP} for any $E_{iMaxP} < 0.874$ Y.

Let us consider a passenger with $E_{iMaxP} = 1000$ hours (uses train transport frequently). Then, the point estimates of the individual risk is $R_{i\phi P} = 1.305 \cdot 10^{-6} < R_{iAccP}$. Hence, the individual risk seems acceptable.

1.2 Imprecision of the original approach

It is crucial to know, whether our point estimate $R_{i\phi P}$ is meaningful or not. We have to find out, if the point estimate $R_{i\phi P}$ approximates the individual risk sufficiently well or not. This validation can be done by constructing the confidence interval (see Figure 1). It is important to mention that the confidence level (e.g. 95%) of a confidence interval expresses the probability which is related to the reliability of the estimation procedure. It is not the probability that a particular confidence interval includes the actual individual risk (see Neyman, 1937).

The one-sided 95% confidence interval for our passengers with $E_{iMaxP} = 1000$ hours accounts for $(0, 9.911 \cdot 10^{-5})$. The right boundary is approximately 76 times greater than the point estimate of the individual risk. Hence, we are facing the situation depicted in the right part of Figure 1.

The confidence interval is overly wide and is not below the threshold. It means, that we cannot say that the real individual risk is acceptable (is below R_{iAccP}). It has to be pointed out that this does not mean that the individual risk is unacceptable, we just cannot make a valid decision. This problem stems from estimating very small probabilities (or risks) which leads to huge inaccuracies (see Figure 5).

1.3 Basic risk – proposed approach

In the previous section, we demonstrated that it is not appropriate to ask, whether the point estimate of the individual risk is below the threshold. This gives us no information on the real individual risk due to overly wide confidence intervals. However, $R_{i\phi P}$ still

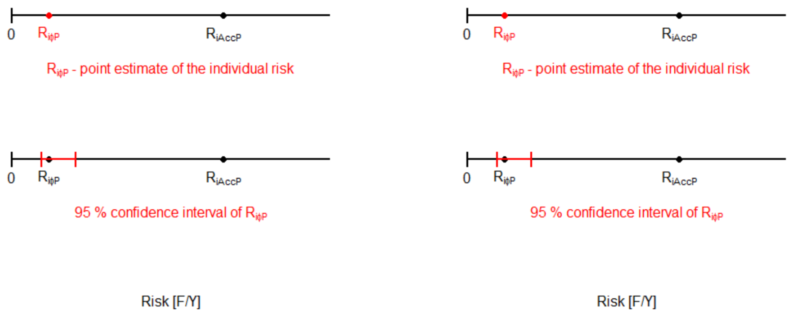

Figure 1: Comparison of a meaningful estimate of $R_{i\phi P}$ (left; narrow confidence interval) and an estimate of $R_{i\phi P}$ which gives us no information on actual individual risk (right; wide confidence interval going far behind the limit R_{iAccP}).

gives us certain information which can be used. Therefore, we suggest to investigate the probability of the individual risk being greater than R_{iAccP} conditioned by the knowledge on $R_{i\phi P}$. In other words, we substitute the question if $R_{i\phi P} < R_{iAccP}$ by asking if $P(\bar{R}_{i\phi P} \geq R_{iAccP}|R_{i\phi P}) < \alpha$ (see Figure 2), where $\bar{R}_{i\phi P}$ stands for the basic individual risk (random variable) and α is the allowed probability of incorrect acceptance (e.g. 5% or 10%).

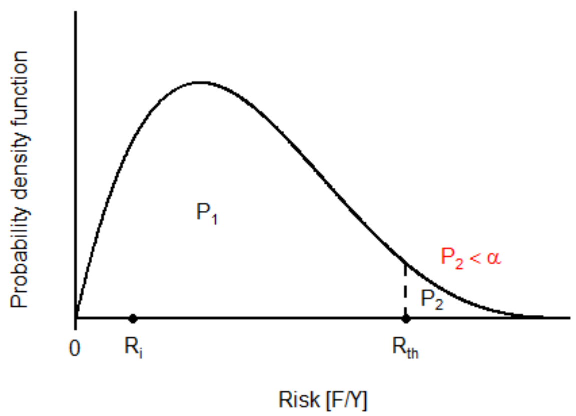

Figure 2: Graphical representation of the proposed approach. Symbol P_1 stands for $P(\bar{R}_{i\phi P} < R_{iAccP}|R_{i\phi P})$ and P_2 denotes $P(\bar{R}_{i\phi P} \geq R_{iAccP}|R_{i\phi P})$.

The probability $P(\bar{R}_{i\phi P} \geq R_{iAccP}|R_{i\phi P})$ can be calculated by the use of Bayes' theorem (see MacKay, 2003). The inputs of this computation are:

E_P	=	$6.4 \cdot 365 \cdot 7870134$ min
$R_{c\phi}$	=	0.4 F/Y
E_{iMaxP}	=	1000 hours
$R_{i\phi P}$	=	$(R_{c\phi} E_{iMaxP})/E_P$

In our case for $E_{iMaxP} = 1000$, we use formula (4) and get 0.41%. It means that we can be quite sure regarding our decision to accept the basic risk for an individual person. The dependence of $P(\bar{R}_{i\phi P} \geq R_{iAccP} | R_{i\phi P})$ on the point estimate of the individual risk is shown in Figure 6.

1.4 Case-specific risk – original approach

The case-specific risk is connected to a particular project under assessment. Assume, that the case-specific collective risk R_{cCase} caused by realization of the project and the operation parameters are known. Furthermore, the expected maximum occurrence of similar projects (projects without any relation to assessed project) can be estimated within an average travel distance of a passenger as N_{OC}. The original approach of FOT counted with an average travel distance of 50.5 km which is connected, however, to the largest Swiss railway company SBB (Swiss Federal Railways) only. Considering the whole railway network of Switzerland (including narrow gauge railways) a significantly shorter average travel distance of 34.7 km should be used. The average travel distance was obtained by dividing the number of total person-kilometers of $2 \cdot 10^{10}$ Pkm/Y by the total number of passenger journeys accounting for $576 \cdot 10^6$ journeys/Y.

The aim is to decide, whether the overall individual risk still remains below the threshold. In Slovak et al. (2012), page 6, the overall individual risk is estimated by a sum of point estimates of the basic individual risk and the case-specific individual risk. Although this procedure is correct, it includes establishing a sufficient margin by setting N_{OC} unnecessarily high. In addition, number of individual passenger's journeys is supposed to be directly connected to the period of exposure of an individual in Slovak et al. (2012). This assumption is not necessary and not valid in general. Furthermore, we already know that point estimates of the individual risks (basic as well as the case-specific risk) do not represent the actual risks accurately. Hence, the sum of the individual risks is even more inaccurate and an approach based solely on the point estimates is therefore not sufficient for individual risk assessment.

1.5 Case-specific risk – proposed approach

Proceeding similarly as in the previous chapter, we calculate the probability that the overall individual risk is greater than $R_{iAccP} = 1 \cdot 10^{-5}$ conditioned by the knowledge on the point estimates of both basic and case-specific individual risks. It means that we compute $P(\bar{R}_{i\phi P} + \bar{R}_{iCase} \geq R_{iAccP} | R_{i\phi P}, R_{iCase})$, where $\bar{R}_{iCase}$ stands for the case-specific individual risk (random variable) and R_{iCase} is its point estimate which is calculated by the following formula

$$R_{iCase} = W_{Case} N_{OC} (R_{cCase} n_{iCase})/N_{PCase}, \tag{2}$$

where N_{PCase} and n_{iCase} are the number of passengers' journeys and the number of individual passenger's journeys, respectively, which pass the case-specific risk. N_{OC} stands for the expected maximum occurrence of similar projects within an average track and $W_{Case}$is the predicted frequency of repetition of the case-specific risk under investigation during a passengers average journey. We accept the overall individual risk, if this probability is sufficiently small (lower than α).

For instance, collective risk caused by adaptation of interlocking functionality with regard to increase of station track speed was estimated and the operation parameters are known ($R_{cCase} = 8.3 \cdot 10^{-4}$ F/Y, 160 trains with passengers per day, 140 passengers per train, $N_{OC} = 10$ and $W_{Case} = 1$). $P(\bar{R}_{i\phi P} + \bar{R}_{iCase} \geq R_{iAccP} | R_{i\phi P}, R_{iCase})$ can be calculated by the use of Bayes' theorem, similarly as in the previous chapter. The inputs of this calculation are:

E_P	=	$6.4 \cdot 365 \cdot 7870134$ min
$R_{c\phi}$	=	0.4 F/Y
E_{iMaxP}	=	1000 hours
$R_{i\phi P}$	=	$(R_{c\phi} E_{iMaxP})/E_P$
N_{PCase}	=	$160 \cdot 140 \cdot 365$ journeys / Y
R_{cCase}	=	$8.333 \cdot 10^{-4}$ F/Y
W_{Case}	=	1
n_{iCase}	=	500 journeys / Y
R_{iCase}	=	$W_{Case} N_{OC} (R_{cCase} n_{iCase})/N_{PCase}$ F/Y

According to formula (5), we arrive at 0.98% (when $E_{iMaxP} = 1000$ hours and $n_{iCase} = 500$ journeys) probability of incorrect acceptance of the overall individual risk. Therefore, we are still quite sure in concern of our decision to accept the overall individual risk. The dependence of $P(\bar{R}_{i\phi P} + \bar{R}_{iCase} \geq R_{iAccP} | R_{i\phi P}, R_{iCase})$ on R_{iCase} for $R_{i\phi P} = 1.305 \cdot 10^{-6}$ (it corresponds to $E_{iMaxP} = 1000$ hours) is shown in Figure 7.

We remark that it is not necessary to specify the threshold for the case-specific risk, because we are interested in the overall individual risk, not only in the case-specific risk.

1.6 Comparison of the original and the proposed approach

The proposed approach extends the original framework by computing the probability that the individual risk is not below the threshold conditioned by the knowledge of the point estimate of the individual risk (see Figure 3).

Let us compare the original and the proposed approach of the overall individual risk assessment on the following actual examples.

Example 1

Collective risk caused by adaptation of interlocking functionality with regard to increase of station track speed was estimated and the operation parameters are known. In the original setting, we have the following inputs:

E_P	=	$6.4 \cdot 365 \cdot 7870134$ min
$R_{c\phi}$	=	0.4 F/Y
E_{iMaxP}	=	1000 hours $= 0.114$ Y
N_{PCase}	=	$160 \cdot 140 \cdot 365$ journeys / Y = $8.176 \cdot 10^6$ journeys / Y
R_{cCase}	=	$8.333 \cdot 10^{-4}$ F/Y
N_{OC}	=	10 projects
n_{iCase}	=	$1000/(31.9/60) = 1881$ journeys / Y

According to (1) and (2), we compute the point estimates of basic and case-specific individual risks. We arrive at $R_{i\phi P} = 1.305 \cdot 10^{-6}$ and $R_{iCase} = 1.917 \cdot 10^{-6}$.

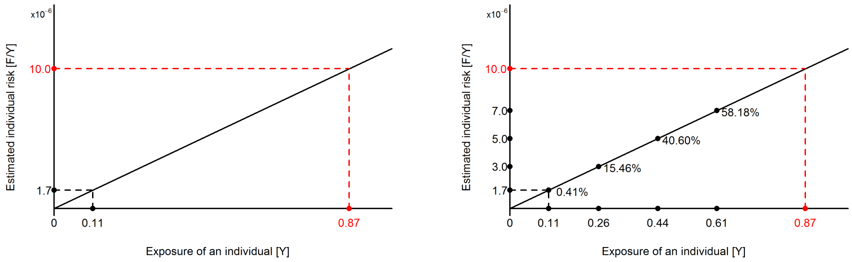

Figure 3: Graphical representation of the original approach (left) and the proposed approach (right) applied to the basic individual risk analysis. $R_{i\phi P} = R_{iAccP} = 1 \cdot 10^{-5}$ F/Y and its corresponding period of exposure ($E_{iMaxP} = 0.87$ Y) are highlighted by red color. Percentages in the proposed approach express the probability that the individual risk is not below the threshold conditioned by the knowledge of $R_{i\phi P}$.

Hence, $R_{i\phi P} + R_{iCase} = 0.322 \cdot 10^{-5} < 1 \cdot 10^{-5} = R_{iAccP}$ and according to the original approach, we accept the overall individual risk. However, the probability that the actual individual risk is greater than R_{iAccP} is 13.59% in this situation. We can see this probability as excessively high and therefore do not accept the overall risk. In other words, the proposed approach showed us that our acceptance based only on the point estimates of the individual risks could be wrong.

On the other hand, inputs N_{OC} and n_{iCase} were originally set with a sufficient safety margin to cover the inaccuracies in calculations. This margin is no longer needed and we can use more realistic estimation of FOT of $N_{OC} = 3$ and $n_{iCase} = 500$, for instance. According to formula (5) as in section 1.5, we arrive at 0.54% probability that the actual individual risk is greater than R_{iAccP}. Hence, the proposed approach gives us a good reason to accept the individual risk.

Example 2

Pendolino trains have higher risk of derailment in comparison with conventional trains due to the higher speed and tilting coaches. The collective risk of derailment of a pendolino train was estimated by SBB as $2.169 \cdot 10^{-8}$ per average distance traveled (34.7 km). Because the collective risk was specified per average distance traveled (not per year), we take the average number of passengers in a pendolino train instead of the total number of journeys per year. The average number of passengers in a pendolino train is 200. The inputs of our calculations are:

$$
\begin{aligned}
E_P &= 6.4 \cdot 365 \cdot 7870134 \text{ min} \\
R_{c\phi} &= 0.4 \text{ F/Y} \\
E_{iMaxP} &= 1000 \text{ hours} = 0.114 \text{ Y} \\
N_{PCase} &= 200 \text{ passengers} \\
R_{cCase} &= 2.169 \cdot 10^{-8} \text{ F/Y} \\
N_{OC} &= 10 \text{ projects} \\
n_{iCase} &= 1000/(31.9/60) = 1881 \text{ journeys / Y}
\end{aligned}
$$

Variable n_{iCase} can be viewed as the number of average journeys traveled in a pendolino train by an individual during a year. According to (1) and (2), we compute the point estimates of basic and case-specific risks. We arrive at $R_{i\phi P} = 1.305 \cdot 10^{-6}$ and $R_{iCase} = 2.040 \cdot 10^{-6}$. Hence, $R_{i\phi P} + R_{iCase} = 0.335 \cdot 10^{-5} < 1 \cdot 10^{-5} = R_{iAccP}$ and according to the original approach, we accept the overall individual risk. However, the probability that the actual individual risk is greater than $1 \cdot 10^{-5}$ is excessively high. It accounts for 15.57%.

As mentioned in the previous example, inputs N_{OC} and n_{iCase} were originally set with a sufficient safety margin to cover the inaccuracies in calculations. Thanks to the proposed method, we can renounce on the conservative safety margin and we set $N_{OC} = 3$ and $n_{iCase} = 500$ (i.e. approximately 266 hours in pendolino trains during a year), for instance. According to formula (5), we arrive at 0.55% probability that the actual individual risk is greater than $1 \cdot 10^{-5}$. Hence, we can accept the individual risk likewise as by the us of the original approach.

2 Practical use of the proposed approach

2.1 Checking the confidence of the estimated individual risk

Confidence of point estimates of individual risk has to be considered. We showed in previous sections that it cannot be done by the use of confidence intervals due to huge uncertainty. Therefore, we proposed an approach based on the Bayes' theorem which can be applied for any particular values of the basic individual risk and case-specific individual risk. The computation provides the probability that the overall individual risk is higher than the threshold for the risk acceptance observing $R_{i\phi P}$ and R_{iCase} (uncertainty of the risk acceptance). It depends on particular users how large uncertainty is acceptable for them. We recommend to set the threshold between 5% to 10%.

Although this approach provides a statistically accurate answer on the acceptability of the individual risk, it can be viewed as rather complicated to use by practitioners due to computations of integrals in formulas (4) and (5). This issue can be solved by computing threshold values of R_{iCase} for fixed $R_{i\phi P}$ and various α in advance (see Table 2 and Figure 4).

Table 2: The table of acceptance for $R_{i\phi P} = 1.305 \cdot 10^{-6}$ and $E_{iMaxP} = 1000$ hours.

α	5%	10%	20%	30%
R_{iCase} [F/Y]	$1.233 \cdot 10^{-6}$	$1.666 \cdot 10^{-6}$	$2.303 \cdot 10^{-6}$	$2.905 \cdot 10^{-6}$

Decision-making on acceptability of a particular case-specific individual risk consists of computing its point estimate R_{iCase} and comparing it with values in a table of acceptance (Table 2). Therefore, the proposed approach does not imply any additional effort for a user in comparison with the current approach.

2.2 Recommendations for the individual risk assessment procedure

The adopted procedure for the assessment of a particular case-specific individual risk considering all findings of the methodological evaluation consists from the following two steps:

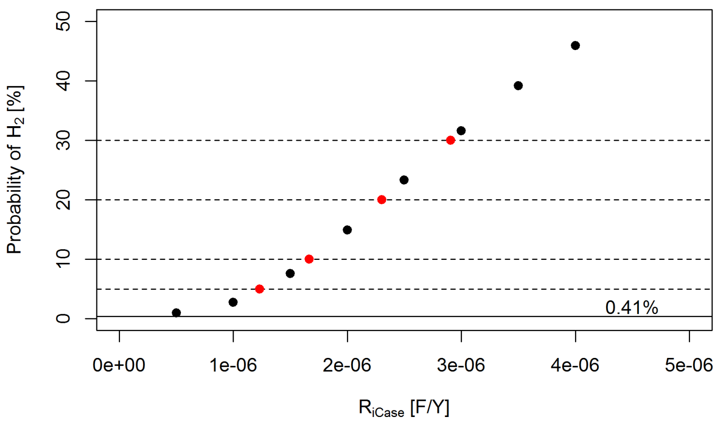

Figure 4: Threshold values of R_{iCase} for fixed for $R_{i\phi P} = 1.305 \cdot 10^{-6}$, $E_{iMaxP} = 1000$ hours and various α highlighted by red color (see Table 2 for exact values).

1. computing the point estimate R_{iCase} of the project specific individual risk according to the formula:

$$R_{iCase} = W_{Case} N_{OC} (R_{cCase} n_{iCase})/N_{PCase},$$

where

N_{PCase}	–	cumulative number of passengers' journeys which pass the case-specific risk
R_{cCase}	–	case-specific collective risk
N_{OC}	–	expected periodicity of similar projects within an average track
W_{Case}	–	predicted frequency of repetition of the case-specific risk under investigation during a passengers average journey
n_{iCase}	–	number of individual passenger's journeys which pass the case-specific risk
R_{iCase}	–	point estimate of the case-specific individual risk

2. comparing the point estimate R_{iCase} with values in a table of acceptance (Table 2 computed for $R_{i\phi P} == 1.305 \cdot 10^{-6}$ F/Y). Aiming on the risk acceptance uncertainty below 10%, the point estimate of the case-specific individual risk would have to be lower than the $1.667 \cdot 10^{6}$ F/Y.

Reducing the proposed approach into these two steps allows practitioners to effectively apply the proposed approach and does not imply any additional effort for users in comparison with the original approach.

Conclusion

We evaluated the current approach for individual risk assessment used by FOT. This approach was tested and certain improvements were recommended. The point estimates of the individual risk calculated according to formulas in Slovak et al. (2012) are correct. However, it was shown that comparing the point estimate of the individual risk with the threshold is misleading due to enormous uncertainty in estimating extremely small probabilities. We do not recommend using the original approach based on the point estimates exclusively, because it can lead to the acceptance of the individual risk when the probability, that the individual risk is unacceptable, is excessively high.

Decision on the acceptability of the individual risk should be made under more precise information concerning the actual individual risk. The point estimate of the individual risk is not sufficient to make such a decision. Therefore, we introduced a new approach based on the Bayes' theorem. Our approach provides a reasonable and accurate answer on the acceptability of the individual risk and is therefore more appropriate for decision-makers than the original approach. In addition, our approach enables more realistic setting of parameters (e. g. n_{iCase} and N_{OC}).

All the above-mentioned procedures can be programmed into a stand-alone application. We therefore also suggest to prepare such an application to facilitate the work of decision-makers. The inputs and outputs of the software realization were outlined in sections A.1.3 and 2.

Calculations based on formulas (4) and (5) are stable with regard to the changes of input variables (see Figures 6 and 7). Hence, also the whole process of decision-making depends continuously on the input variables.

Besides the risk of death also other types of risks should be considered (i. e. the risk of severe injury and the risk of light injury). These risks can be assessed similarly as the risk of death. Afterwards, the probabilities of incorrect acceptance of the risks can be combined to get the probability that at least one risk is accepted incorrectly.

We worked with average values (e.g. average number of trains, average number of passengers per train, average journey) in this study. However, risks induced by railway transport are spatially dependent (certain tracks can be more risky than others). For instance, an individual passenger can travel only along more (or less) risky tracks when commuting to work. Therefore, we suggest to take into account also the spatial component of the risk. This issue remains for further investigation.

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A Appendices

A.1 Statistical details of the proposed approach

Statements and conclusions about the methods presented in the first section are justified in this section from the statistical point of view. First, basic assumptions are made, stochastic model is described and formula (1) for computing the point estimate of the basic individual risk is derived (section A.1.1). In section A.1.2, the interval estimate of the individual risk is described. Afterwards, the proposed approach is described in section A.1.3.

A.1.1 Derivation of the formula for individual risk calculation

Assume that the point estimate of the collective risk ($R_{c\phi}$ [F/Y]) and the cumulative period of exposure (E_P [Y]) are known from the empirical data. First, let us derive the formula (2) from Slovak et al. (2012). We denote:

n – total number of passengers during a year (unknown in general),
X_i – Bernoulli distributed random variable with parameter p_i, $i = 1, \dots, n$; $X_i = 0$ if passenger i did not die, $X_i = 1$ if passenger i died, p_i is the probability of death of passenger i,
S – the sum of X_i; S stands for the number of fatalities; it is a random variable following Poisson Binomial distribution.

The expected value of X_i is the individual risk ($R_{i\phi P}$). Hence, $R_{i\phi P} = \mathrm{E}X_i = p_i$. Similarly, the expected value of S is the collective risk ($R_{c\phi}$). Supposing the independence between X_i, it holds that

$$R_{c\phi} = \mathrm{E}S = \sum_{i=1}^{n} \mathrm{E}X_i = \sum_{i=1}^{n} R_{i\phi P}$$

and the variance of S equals to

$$\sum_{i=1}^{n} p_i(1 - p_i) = \sum_{i=1}^{n} R_{i\phi P}(1 - R_{i\phi P}).$$

Let us suppose that $R_{i\phi P}$ depends on the time of exposure (E_{iMaxP}) linearly. It means that $R_{i\phi P} = \alpha E_{iMaxP}$ for some unknown real number $\alpha > 0$. We know that

$$R_{c\phi} = \sum_{i=1}^{n} R_{i\phi P} = \sum_{i=1}^{n} \alpha E_{iMaxP} = \alpha \sum_{i=1}^{n} E_{iMaxP} = \alpha E_P.$$

Hence, $\alpha = R_{c\phi}/E_P$ and thus we arrive at the formula (1).

Now, there is no need to put $E_{iMaxP} = 1$ [Y] and turn the relation (1) to the formula (3) from Slovak et al. (2012). Better approach would be to look at E_{iMaxP} as an independent variable taking values, for example, from $\langle 0{,}1000 \rangle$ [hours].

Overall, the formula (1) is well supported by statistics, if it is possible to make assumptions described above. However, we have to keep in mind that $R_{i\phi P}$ is only the point estimate of the real individual risk, as well as $R_{c\phi}$ is the point estimate of the real collective risk.

Point estimates can be sometimes meaningless. The main reason for this is the lack of the data. In our case, we are estimating very small probabilities. Hence, the number of records needed for sufficiently precise estimate is enormously huge (see Figure 5).

For instance, let us suppose that we want to estimate the probability of meeting a person taller than two meters. We randomly choose a thousand people, measure them and find out that one of them is taller than two meters. Hence, the point estimate of the probability of meeting a person taller than two meters is $1 \cdot 10^{-3}$. However, the 95% confidence interval of this estimate is $(0.025 \cdot 10^{-3}, 5.559 \cdot 10^{-3})$.

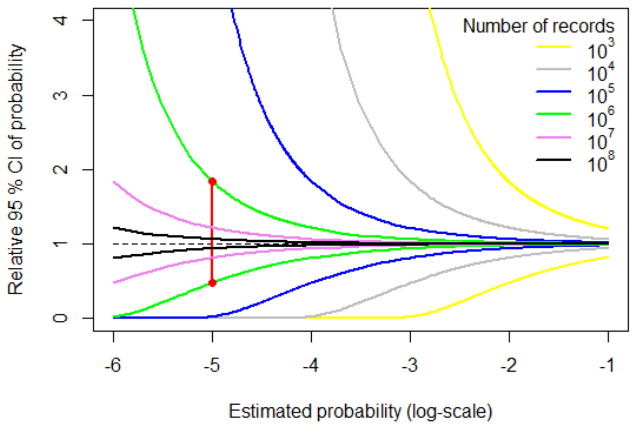

Figure 5: Vertical axis depicts 95% confidence intervals of the estimated probability divided by the estimated probability for different number of records. Logarithmic scale means that the estimated probability is ten to the power of a particular number on the horizontal axis. For instance, 95% confidence interval for probability accounting for $1 \cdot 10^{-5}$ estimated from 10^6 records is highlighted by red color.

The accuracy of the point estimate can be improved by using more data. We can randomly choose ten thousand people (let us assume that ten of them are taller than two meters). The point estimate is still the same, but 95% confidence interval shrinks to $(0.480 \cdot 10^{-3}, 1.838 \cdot 10^{-3})$.

In our case, we are working with probabilities around or below $1 \cdot 10^{-5}$. Furthermore, we are already using as much data as possible (all passengers). Thus, we cannot improve the accuracy of the point estimate of the individual risk by using more data. On the other hand, working with small probabilities has also an advantage. In general, we cannot express a risk, which is composed of two particular risks, as a sum of that two particular risks. However in our case, we can write

$$R = 1 - (1 - R_1)(1 - R_2) = R_1 + R_2 - R_1R_2 \approx R_1 + R_2$$

because R_1R_2 is negligible in comparison with R_1 and R_2 (e.g. if $R_1 = R_2 = 1 \cdot 10^{-6}$, then $R_1R_2 = 1 \cdot 10^{-12}$). This reasoning simplifies some calculations in the following sections.

A.1.2 Interval estimates of the individual risk

It is crucial to know, whether our point estimates $R_{i\phi P}$ is meaningful or not. We can compute standard deviations for random variables X_i to assess this issue. It holds that

$$\mathrm{sd}X_i = \sqrt{p_i(1-p_i)} = \sqrt{R_{i\phi P}(1-R_{i\phi P})}, \tag{3}$$

because X_i is distributed according to the Bernoulli distribution with parameter p_i. We know that $R_{i\phi P}$ is very small number (see Examples 1 and 2). Hence $(1 - R_{i\phi P})$ is almost one and the standard deviation of X_i is, therefore, approximately $\sqrt{R_{i\phi P}}$. Thus, $\mathrm{sd}X_i$ is much greater than $R_{i\phi P}$. Let us demonstrate how large is this difference and how uncertain are our point estimates of the individual risk by constructing the interval estimates of the individual risk in the following examples.

Example 3
Let us suppose that a particular passenger was exposed for $E_{iMaxP} = 1000$ hours. Hence, the point estimate of the individual risk is $R_{i\phi P} = (R_{c\phi}E_{iMaxP})/E_P = 1.305 \cdot 10^{-6}$.

Usually, the Normal distribution would be used to get 95% confidence interval of the parameter of the Binomial distribution ($R_{i\phi P}$ in our case). However, the asymptotic property of the Binomial distribution cannot be used here because of the low value of $R_{i\phi P}$. Therefore, we constructed a Clopper-Pearson interval (see Clopper et al., 1934). Finally, we arrived at the one-sided 95% confidence interval of the individual risk. It accounts for

$$(0, 9.911 \cdot 10^{-5}).$$

The right boundary is approximately 76 times greater than the point estimate of the individual risk. Hence, the 95% confidence interval of the individual risk is relatively wide in comparison to the point estimate of the individual risk. Therefore, $R_{i\phi P}$ gives us rather poor estimate of the real individual risk. Furthermore, the confidence interval is not below the threshold R_{iAccP} and we cannot make a decision on acceptability of the basic individual risk.

Example 4
Let us take a passenger with the period of exposure $E_{iMaxP} = 100$ hours. We proceed similarly as in the Example 1 and arrive at $R_{i\phi P} = (R_{c\phi}E_{iMaxP})/E_P = 1.305 \cdot 10^{-7}$ and the 95% confidence interval of the individual risk accounting for

$$(0, 9.991 \cdot 10^{-6}).$$

Likewise as in the Example 1, the 95% confidence interval of the individual risk is relatively wide in comparison to the point estimate of the individual risk. However, the confidence interval is below the threshold R_{iAccP}. Thus, we can accept the basic individual risk on the basis of $R_{i\phi P}$ and its confidence interval. Anyway, a slight increase of E_{iMaxP} above 100 hours leads to a confidence interval which crosses the threshold R_{iAccP}.

To sum it up, estimating very small probabilities (or risks) lead to huge uncertainty. Therefore, it is impossible to make a decision regarding the acceptability of the individual risk. In the next section, we present an approach which improves the original method by expressing the probability of incorrect acceptance of the individual risk.

A.1.3 Proposed approach

Let us define the following hypotheses:

H_1: The overall individual risk is below the threshold.
H_2: The overall individual risk is not below the threshold.

Basic risk We assume that the overall individual risk is composed only of the basic individual risk. We proceed as follows:

1. We set the threshold of the overall individual risk (R_{iAccP}).
2. The basic individual risk is estimated by the point estimate ($R_{i\phi P}$).
3. We accept the overall individual risk, if the probability of H_2 conditioned by the knowledge of $R_{i\phi P}$ is sufficiently small.

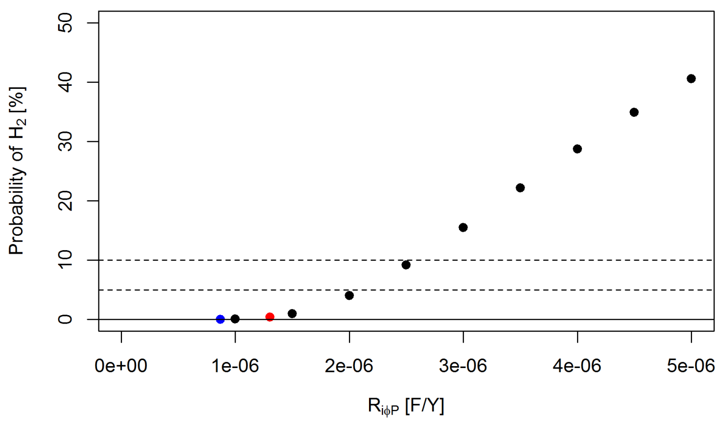

Figure 6: Probability from relation (4) dependent on $R_{i\phi P}$. Red and blue points highlight resulting probabilities for individual risks accounting for $1.305 \cdot 10^{-6}$ and $0.870 \cdot 10^{-6}$, which correspond to the individual period of exposure $E_{iMaxP} = 1000$ hours and the basic collective risk of 0.4 F/Y (with a margin accounting for two fatalities) and 0.267 F/Y (without any margin), respectively.

Let us suppose that the input values (see the list in section 1.3) are given. We can calculate the probability from the third step in the following way using the Bayes' theorem:

$$P(H_2|R_{i\phi P}) = \frac{P(R_{i\phi P}|H_2)P(H_2)}{P(R_{i\phi P})} = \frac{\int_{R_{iAccP}}^{1} P(R_{i\phi P}|p)P(p)\mathrm{d}p}{\int_0^1 P(R_{i\phi P}|p)P(p)\mathrm{d}p} = \frac{\int_{R_{iAccP}}^{1} P(R_{i\phi P}|p)\mathrm{d}p}{\int_0^1 P(R_{i\phi P}|p)\mathrm{d}p}, \quad (4)$$

where $P(p) \equiv 1$ (the simplest vanilla prior distribution when we know nothing about the data). $P(R_{i\phi P}|p)$ stands for the probability that we get $R_{i\phi P}$ supposing that the actual basic risk is p. For instance, if we set $\alpha = 10\%$, then the acceptable values of $R_{i\phi P}$ are approximately below $2.5 \cdot 10^{-6}$ (see Figure 6).

Case-specific risk We consider that apart from the basic individual risk, also the case-specific risk is involved. We proceed similarly as in the previous case and we add steps to include the case-specific risk:

1. We set the threshold of the overall individual risk (R_{iAccP}).
2. The basic individual risk is estimated by the point estimate ($R_{i\phi P}$).
3. The case-specific individual risk is estimated by the point estimate (R_{iCase}).
4. We accept the overall individual risk, if the probability of H_2 conditioned by the knowledge of $R_{i\phi P}$ and R_{iCase} is sufficiently small.

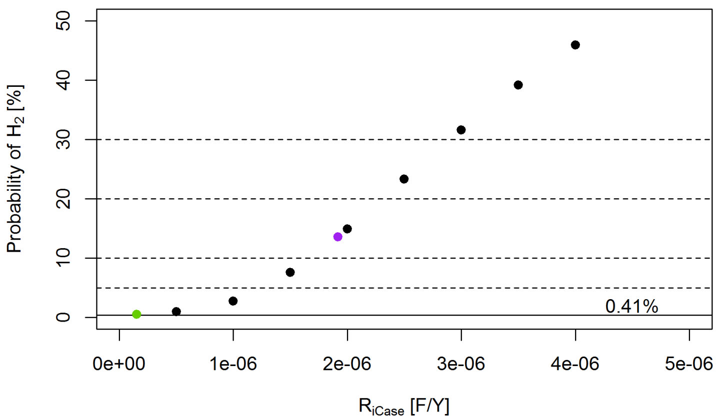

Figure 7: Probability from relation (5) dependent on R_{iCase} when $R_{i\phi P} = 1.305 \cdot 10^{-6}$. Purple and green points highlight resulting probabilities for case-specific individual risk accounting for $1.917 \cdot 10^{-6}$ ($n_{iCase} = 1881$, $N_{OC} = 10$) and $0.153 \cdot 10^{-6}$ ($n_{iCase} = 500$, $N_{OC} = 3$).

Assume that the input values are given (see the list in section 1.5, Example 1). The probability from the last step can be calculated by the use of Bayes' theorem:

$$P(\mathrm{H}_2|R_{i\phi P}, R_{iCase}) = 1 - \frac{P(R_{i\phi P}, R_{iCase}|\mathrm{H}_1)P(\mathrm{H}_1)}{P(R_{i\phi P}, R_{iCase})} =$$
$$= 1 - \frac{\int_0^{R_{iAccP}} \int_0^{R_{iAccP}-p_1} P(R_{i\phi P}|p_1)P(R_{iCase}|p_2)\mathrm{d}p_2\mathrm{d}p_1}{\int_0^1 \int_0^1 P(R_{i\phi P}|p_1)P(R_{iCase}|p_2)\mathrm{d}p_1\mathrm{d}p_2} =$$
$$= 1 - \frac{\int_0^{R_{iAccP}} P(R_{i\phi P}|p_1) \int_0^{R_{iAccP}-p_1} P(R_{iCase}|p_2)\mathrm{d}p_2\mathrm{d}p_1}{\int_0^1 P(R_{i\phi P}|p_1)\mathrm{d}p_1 \int_0^1 P(R_{iCase}|p_2)\mathrm{d}p_2}, \tag{5}$$

where $P(R_{i\phi P}|p_1)$ stands for the probability that we observe $R_{i\phi P}$ supposing that the actual basic risk is p_1 and $P(R_{iCase}|p_2)$ is the probability that we get R_{iCase} supposing that the actual case-specific risk is p_2. The probability from (5) cannot be lower than 0.41% when the individual period of exposure E_{iMaxP} is 1000 hours (see Figure 7).