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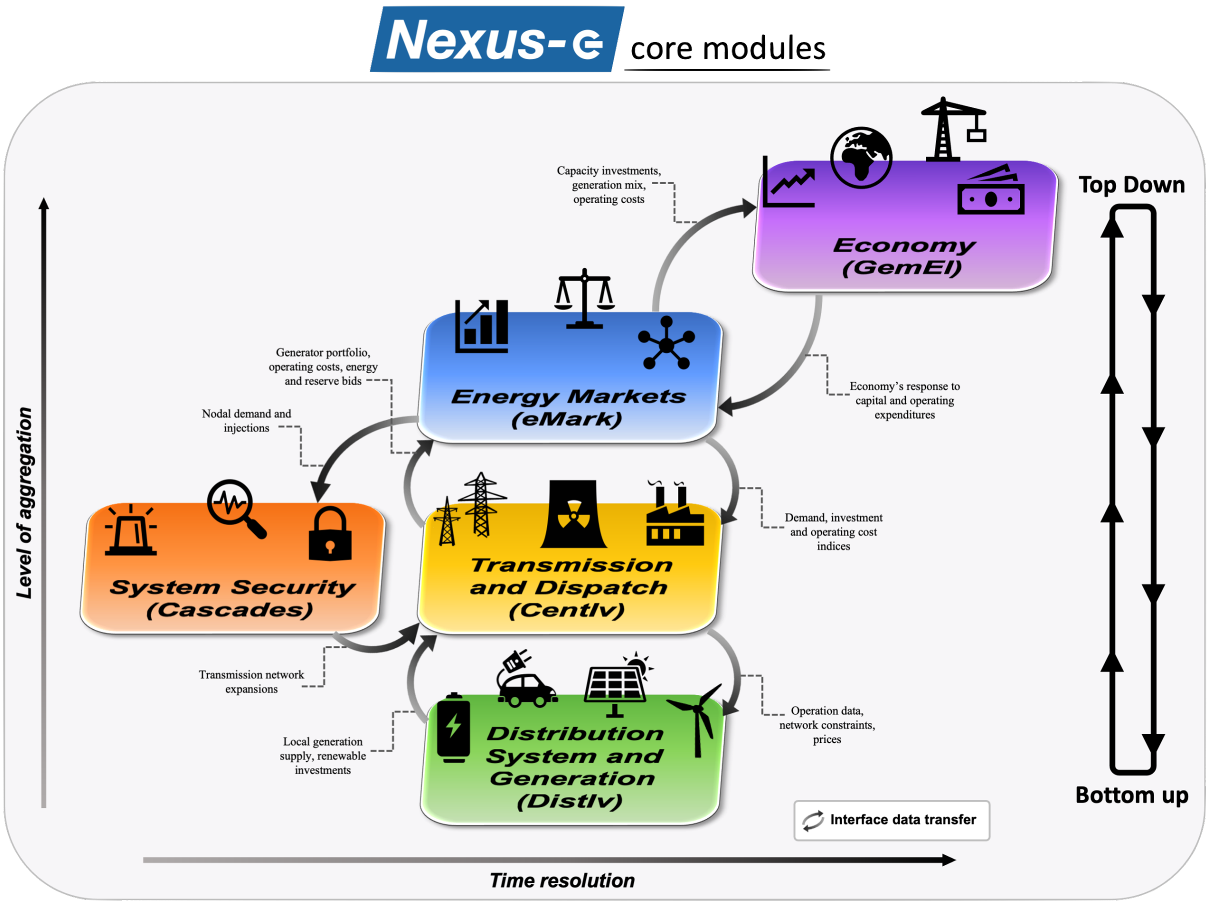
Final report

Nexus-e: Integrated Energy Systems Modeling Platform

GemEl Module Documentation



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Summary

Policy changes in the energy sector result in wide-ranging implications throughout the entire energy system and influence all sectors of the economy. Due partly to the high complexity of combining separate models, few attempts have been undertaken to model the interactions between the components of the energy-economic system. The Nexus-e Integrated Energy Systems Modeling Platform aims to fill this gap by providing an interdisciplinary framework of modules that are linked through well-defined interfaces to holistically analyze and understand the impacts of future developments in the energy system. This platform combines bottom-up and top-down energy modeling approaches to represent a much broader scope of the energy-economic system than traditional stand-alone modeling approaches.

In Phase 1 of this project, the objective is to develop a novel tool for the analysis of the Swiss electricity system. This study illustrates the capabilities of Nexus-e in answering the crucial questions of how centralized and distributed flexibility technologies could be deployed in the Swiss electricity system and how they would impact the traditional operation of the system. The aim of the analysis is not policy advice, as some critical developments like the European net-zero emissions goal are not yet included in the scenarios, but rather to illustrate the unique capabilities of the Nexus-e modeling framework. To answer these questions, consistent technical representations of a wide spectrum of current and novel energy supply, demand, and storage technologies are needed as well as a thorough economic evaluation of different investment incentives and the impact investments have on the wider economy. Moreover, these aspects need to be combined with modeling of the long- and short-term electricity market structures and electricity networks. This report illustrates the capabilities of the Nexus-e platform.

The Nexus-e Platform consists of five interlinked modules:

1. General Equilibrium Module for Electricity (GemEl): a computable general equilibrium (CGE) module of the Swiss economy,
2. Centralized Investments Module (CentIv): a grid-constrained capacity expansion planning module considering system flexibility requirements,
3. Distributed Investments Module (DistIv): a generation expansion planning module of distributed energy resources,
4. Electricity Market Module (eMark): a market-based dispatch module for determining generator production schedules and electricity market prices,
5. Network Security and Expansion Module (Cascades): a power system security assessment and transmission system expansion planning module.

This report provides the description and documentation for the GemEl module, which is utilized in the Nexus-e framework to provide feedback between the economy and the bottom-up models, as well as give insights in the economic effects of the implemented scenarios. GemEl is a recursive-dynamic CGE model with up to 75 sectors and 14 household types. It is based on the 2014 Swiss energy-specific differentiated input–output table for the energy sector (IOT-Energy).

Zusammenfassung

Politische Veränderungen im Energiesektor haben weitreichende Auswirkungen auf das gesamte Energiesystem und beeinflussen alle Sektoren der Wirtschaft. Aufgrund der hohen Komplexität der Energiewirtschaft, wurden bisher nur wenige Versuche unternommen, die Wechselwirkungen zwischen den einzelnen Komponenten dieses Systems zu modellieren. Nexus-e, eine Plattform für die Modellierung von integrierten Energiesystemen, schliesst diese Lücke und schafft einen interdisziplinäre Plattform, in welcher verschiedene Module über klar definierten Schnittstellen miteinander verbunden sind. Dadurch können die Auswirkungen zukünftiger Entwicklungen in der Energiewirtschaft ganzheitlicher analysiert und verstanden werden. Die Nexus-e Plattform ermöglicht die Kombination von „Bottom-Up“ und „Top-Down“ Energiemodellen und ermöglicht es dadurch, einen breiteren Bereich der Energiewirtschaft abzubilden als dies bei traditionellen Modellierungsansätzen der Fall ist.

Phase 1 dieses Projekts zielt darauf ab, ein neuartiges Instrument für die Analyse des schweizerischen Elektrizitätssystems zu entwickeln. Um die Möglichkeiten von Nexus-e zu veranschaulichen, untersuchen wir die Frage, wie zentrale und dezentrale Flexibilitätstechnologien im schweizerischen Elektrizitätssystem eingesetzt werden können und wie sie sich auf den traditionellen Betrieb des Energiesystems auswirken würden. Ziel der Analyse ist es nicht Empfehlungen für die Politik zu geben, da einige wichtige Entwicklungen wie das Europäische Netto-Null-Emissionsziel noch nicht in den Szenarien enthalten sind. Vielmehr möchten wir die einzigartigen Fähigkeiten der Modellierungsplattform Nexus-e vorstellen. Um diese Fragen zu beantworten, ist eine konsistente technische Darstellungen aktueller und neuartiger Energieversorgungs-, Nachfrage- und Speichertechnologien, sowie eine gründliche wirtschaftliche Bewertung der verschiedenen Investitionsanreize und der Auswirkungen der Investitionen auf die Gesamtwirtschaft erforderlich. Darüber hinaus müssen diese Aspekte mit der Modellierung der lang- und kurzfristigen Strommarktstrukturen und Stromnetze kombiniert werden.Dieser Report veranschaulicht die Fähigkeiten der Nexus-e Plattform.

Die Nexus-e Plattform besteht aus fünf miteinander verknüpften Modulen:

1. Allgemeines Gleichgewichtsmodul für Elektrizität (GemEl): ein Modul zur Darstellung des allgemeinen Gleichgewichts (CGE) der Schweizer Wirtschaft,
2. Investitionsmodul für zentrale Energiesysteme (CentIv): ein Modul zur Planung des netzgebundenen Erzeugungsausbaus (GEP) unter Berücksichtigung der Anforderungen an die Systemflexibilität,
3. Investitionsmodul für dezentrale Energiesysteme (DistIv): ein GEP-Modul für dezentrale Energieerzeugung,
4. Strommarktmodul (eMark): ein marktorientiertes Dispatch-Modul zur Bestimmung von Generator-Produktionsplänen und Strommarktpreisen,
5. Netzsicherheits- und Erweiterungsmodul (Cascades): ein Modul zur Bewertung der Sicherheit des Energiesystems und zur Planung der Erweiterung des Übertragungsnetzes.

Dieser Bericht beinhaltet die Beschreibung und die Dokumentation des GemEl-Moduls. Dieses Modul wird im Rahmen der Nexus-e Plattform verwendet, um Feedback zwischen der Wirtschaft und den Bottom-up-Modellen zu liefern und Einblicke in die wirtschaftlichen Auswirkungen der implementierten Szenarien zu geben. GemEl ist ein rekursiv-dynamisches CGE-Modell mit bis zu 75 Sektoren und 14 Haushaltstypen. Es basiert auf der energiespezifischen Schweizer Input-Output-Tabelle (IOT) für 2014.

Résumé

Les changements de politique dans le secteur de l'énergie ont de vastes répercussions sur l'ensemble du système énergétique et influencent tous les secteurs de l'économie. En partie à cause de la grande complexité de la combinaison de modèles séparés, peu de tentatives ont été entreprises pour modéliser les interactions entre les composantes du système économico-énergétique. La plateforme de modélisation des systèmes énergétiques intégrés Nexus-e vise à combler cette lacune en fournissant un cadre interdisciplinaire de modules qui sont reliés par des interfaces bien définies pour analyser et comprendre de manière holistique l'impact des développements futurs du système énergétique. Cette plateforme combine des approches de modélisation énergétique ascendante et descendante pour représenter un champ d'application beaucoup plus large du système économico-énergétique que les approches de modélisation indépendantes traditionnelles.

Dans la phase 1 de ce projet, l'objectif est de développer un nouvel outil pour l'analyse du système électrique suisse. Cette étude sert à illustrer les capabilités de Nexus-e à répondre aux questions cruciales de comment les technologies de flexibilité centralisées et décentralisées pourraient être déployées dans le système électrique suisse et comment elles affecteraient le fonctionnement traditionnel du système. Le but de cette analyse n'est pas d'offrir de conseils politiques, en tant que les scénarios ne considèrent pas des développements critiques comme l'objectif Européen d'atteindre zéro émission nette, mais d'illustrer les capabilités uniques de la plateforme Nexus. Pour répondre à ces questions, des représentations techniques cohérentes d'un large éventail de technologies actuelles et nouvelles d'approvisionnement, de demande et de stockage d'énergie sont nécessaires, ainsi qu'une évaluation économique approfondie des différentes incitations à l'investissement et de l'impact des investissements sur l'économie au sens large. En outre, ces aspects doivent être combinés avec la modélisation des structures du marché de l'électricité et des réseaux d'électricité à long et à court terme. Ce rapport illustre les capacités de la plateforme Nexus-e.

La plateforme Nexus-e se compose de cinq modules interconnectés:

1. Module d'équilibre général pour l'électricité (GemEl) : un module d'équilibre général calculable (CGE) de l'économie suisse,
2. Module d'investissements centralisés (CentIv) : un module de planification de l'expansion de la production (GEP) soumise aux contraintes du réseau, qui tient compte des exigences de flexibilité du système,
3. Module d'investissements distribués (DistIv) : un module GEP de la production décentralisée d'énergie,
4. Module du marché de l'électricité (eMark) : un module de répartition basé sur le marché pour déterminer les calendriers de production des producteurs et les prix du marché de l'électricité,
5. Module de sécurité et d'expansion du réseau (Cascades) : un module d'évaluation de la sécurité du système électrique et de planification de l'expansion du système de transmission.

Ce rapport fournit la description et la documentation du module GemEl, qui est utilisé dans le cadre de Nexus-e pour fournir un retour d'information entre l'économie et les modèles ascendants, ainsi que pour donner un aperçu des effets économiques des scénarios mis en œuvre. GemEl est un modèle EGC récursif-dynamique comprenant jusqu'à 75 secteurs et 14 types de ménages. Il est basé sur le tableau entrées-sorties (TES) spécifique à l'énergie suisse de 2014.

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Abbreviations

BAU	business-as-usual
Cascades	Network Security and Expansion Module
CentIv	Centralized Investments Module
CES	constant-elasticity of substitution
CGE	computable general equilibrium
CHP	combined heat and power
CO_2	carbon dioxide
CP	complementarity problem
DistIv	Distributed Investments Module
eMark	Electricity Market Module
EMP-E	Energy Modeling Platform for Europe
EU	European Union
GDP	gross domestic product
GemEl	General Equilibrium Module for Electricity
HBS	household budget survey
IOT-Energy	differentiated input–output table for the energy sector
MCP	mixed-complementarity problem
MSW	municiple solid waste
MWh	megawatt hour
OM	operation and maintenance
PV	photovoltaic
ROW	rest of the world
TWh	terawatt hour

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1 Introduction

1.1 Module purpose

The GemEl module (Applied General Equilibrium of the Economy, the Environment, and Energy) is a detail-rich CGE model for Switzerland based on the most actual economic data available. The model simulates the markets for all goods and services produced and demanded. It can be used for almost any policy measure and especially for evaluating the efficiency and distributional effects of energy policy measures as well as new investments in new electricity generation. Analysis of distributional effects is possible because the model contains 14 active and retired household groups distinguished by income. The model also can keep track of emissions and the yearly produced and demanded electricity.

1.2 Process overview

The GemEl module simulates the Swiss economy with over 77 sectors and 14 household groups using a yearly resolution. All good and factors markets are cleared simultaneously as GemEl is formulated as a system of non-linear equalities and inequalities resulting in the prevailing for each good the market prices, as well as the demand and supply.

1.3 Attributes

GemEl can be characterized by:

- Richness in detail: Around 75 sectors producing goods and services, 14 household groups. It also contains the Swiss emission trading system for carbon dioxide (CO_2).
- Dynamics: The model can be run in yearly or multi-yearly periods and is calibrated to the Energy Perspectives growth paths for gross domestic product (GDP), and energy. The total and employed population is updated yearly according to the population scenarios of the Swiss Federal Statistics Office.
- Energy market: Not only the value of electricity demanded or produced but also the quantity (measured in terawatt hour (TWh) per year).
- Environmental indicators: Information on the level of CO_2 emissions in Switzerland.
- Actual Data: The model uses the most recent actual data: the just-released energy- and transport-specific IOT-Energy, the newest household budget survey (HBS) data as well as the electricity statistic for Switzerland.
- Flexibility: It can be easily updated if new data is available or extended if more energy carriers are added to the Nexus-e Framework.

1.4 Capabilities

GemEl can analyze the economic impact of energy policies and changes in the electricity generation mix. It gives information on the behavior of key macroeconomic variables like GDP, ex- and imports, tax revenue, as well as information on changes in sectoral prices and production. On the demand side, GemEl provides information on the distributional impact of energy policies: Which household gains, which household looses? What are the costs for the household of a certain energy policy? Although

GemEI is not a model that produces projections, it can be used to compare alternative energy policies and rank them according to their impact on the efficiency of the economy. As the model has information on the sectoral and demand-side emissions of CO_2, the model can be used to find the CO_2 price necessary for reaching the targets set by the Federal Council.

1.5 Limitations

GemEI has some limitations: It assumes perfect competition on all markets. It has a yearly time resolution contrary to the high resolution of the bottom-up modules of the Nexus-e framework and can, therefore not depict the daily and seasonal price changes. GemEI is a single-country model and therefore can only set policy-induced changes in import prices endogenously.

1.6 Inputs and outputs

Tables 1 and 2 below lists this module's required input and resulting output data. Those data that are input from or sent to another module through an interface are noted with an asterisks (*).

Table 1: Listing of required input data for the GemEI module

Inputs (all yearly and for Switzerland)	**Unit**	**Source**
Input-output-table 2014	CHF	Nathani et al. (2019)
Household survey data 2012-2014	CHF	Bundesamt für Statistik (2019a)
Elasticities	no dimension	taken from several studies cited in the report
Macroeconomic data like GDP, etc.	CHF	Bundesamt für Statistik (2019b)
Energy inputs	Joules, kWh	Swiss Federal Office of Energy (2019), Prognos (2012)
CO_2-Emissions	tonnes	Federal Office for the Environment (2019)
Population and employment	no dimension and full-time equivalents	Bundesamt für Statistik (2015)
Electricity supply*, imports* and exports*	TWh	DistIv, eMark
Costs of generation*, imports and exports*, investments in new generation*	CHF	CentIv, DistIv

Table 2: Listing of resulting output data for the GemEI module.

Output (all yearly, for Switzerland)	**Units**
GDP, exports, imports, sectoral production, tax revenue	CHF or EUR
Welfare and distributional effects	percentage change
Sectoral prices and production, cost indices*	indexed CHF
CO_2 price of permits and CO2 tax*	CHF
Electricity demand*	TWh
CO2-Emissions	tonnes

2 Related work and contributions

GemEl is a CGE model and distinguishes itself from other models in Switzerland through its higher complexity and use of the most recent actual IOT-Energy data. There are several research groups, as well as a consultancy firm currently also working on CGE modeling (listed in Table 3).[1]

Table 3: Actual CGE models used in Switzerland

Model name	Institute	Regions	Dynamics	Used in
CEPE-HH	Chair of Economics / Energy Economics	Switzerland	static	2019
CITE	Chair of Economics / Resource Economics	Switzerland	static	2019
CEPE-316	Chair of Economics / Energy Economics	Global	recursive dynamics	2019
GEMINI-E3	EPFL	Global	static	2019
SWISSGEM-E	Ecoplan	Switzerland	static	2015

Landis et al. (2019) compare several Swiss CGE models used for assessing the economic and technological consequences of reaching emission reduction targets for 2050 in the context of Switzerland:

- The CEPE-HH (Computable General Equilibrium Model for Energy Policy and Economics—with a focus on household consumption) is a static small open economy model of the Swiss economy (Landis, 2019) and designed to assess the implications of environmental and energy regulation in particular on the consumption and welfare of different Swiss households.
- The CEPE-316 is a recursive-dynamic, multi-country model with three regions and 16 sectors. The three regions are Switzerland, the European Union (EU), and the rest of the world (ROW). It is based on the GTAP Power data set 9.1(Aguiar, Narayanan, and McDougall, 2016), which contains detailed information on energy sectors and emissions for the year 2011 (Peters, 2016). The model assumes myopic foresight. Capital stock in the next period is calculated using the actual investments and the depreciated capital at the end of the actual period. The sectoral differentiation is geared to energy questions. The energy sectors are oil, gas, electricity generation (split into peak- or base-load generation from nuclear, coal, gas, wind, hydropower, oil, other energy, and solar), and electricity distribution. The other sectors are transport and the primary, secondary, and tertiary (minus transport) sectors. The transport sector is modeled as most other sectors in the model and does not contain detailed information on transport technologies used. Transportation demand is assumed to grow with the steady-state growth rate without taking into account the move from fossil fuels to electric driven cars. The model allows for exogenous technological change, capacity limits on electricity generation, and inclusions of new generation technologies. One advantage of the model is the multi-regional character, which allows for a more realistic implementation of different emission trading systems, regional endogenous or exogenous energy, or CO_2 taxes compared to a single-country model. Furthermore, the consideration of time allows analyzing changes during the transition period of a policy. The advantage of the use of the GTAP 9 Power data is the detailed representation of the electricity market.
- The CITE (Computable Induced Technical change and Energy) model is a dynamic small open economy model of the Swiss economy with fully endogenous growth. The main feature of CITE is that growth in the different sectors is driven by an expansion in the types of intermediate goods (machines), in accordance with the seminal contribution of Romer (1990). Investments in physical capital and knowledge extend the number of capital varieties, which fosters factor productivity. The CITE model represents different sectors of the economy, including ten non-energy sectors and the electricity sector. Transport is modeled as a non-energy sector without technological detail. Generation technologies are divided into three categories: intermittent technologies including wind

[1] It must be noted that with the exception of the CITE and GEMINI-E3 models, the author either built the model (CEPE-316) or played a role in developing it.

and solar, nuclear power (available until 2034), and constant electricity supply technologies including hydropower, conventional thermal plants, electricity from waste, and biomass (Bretschger and Zhang, 2017) The trade-offs between and within groups are modeled with constant-elasticity-of-production functions. In CITE, a representative consumer allocates income between consumption and investments to maximize its inter-temporal utility under perfect foresight.

- GEMINI-E3 (General Equilibrium Model of International National Interactions between Economy, Energy, and the Environment) is a multi-country, multi-sector, recursive CGE model (Bernard and Vielle, 2008). It is a global model built using the GTAP database and the Swiss IOT-Energy. GEMINI-E3 is recursive dynamic, with backward-looking (adaptive) expectations. In this model, periods are linked through endogenous real interest rates that equate savings and investment. Capital is not mobile across regions. National and regional models are linked by endogenous real exchange rates resulting from constraints on foreign trade deficits or surpluses. It includes a specific representation of the road transport sector, where several types of vehicles are detailed according to the fuel used.
- SWISSGEM-E is a single-country static CGE model for Switzerland using data for the year 2008 developed by Ecoplan (Böhringer and Müller, 2014). It contains 62 sectors, and 15 different types of households. It contains a bottom-up formulation of electricity production. The model has been used to study the impacts of energy and CO_2 taxes.

GemEI distinguishes itself in several aspects and shows a higher complexity as it combines aspects of these models. At the moment, it is the only CGE model in Switzerland based on the most recent actual data (the energy-specific IOT-Energy from 2014 (Nathani et al., 2019)). Furthermore, it makes a clear distinction between the several sources of electricity generation by using actual prices and quantities in TWh (and not, as in all the other models, a unitary benchmark price for all electricity flows). It is also the only CGE model that is developed in close cooperation with engineers. GemEI is also dynamic and it contains several households.

3 Detailed description of the module

The standard CGE model is based on the work of Arrow and Debreu (1954), who proved that, under very general conditions, an equilibrium for the economic system as developed by Walras in 1924 exists.[2] This general equilibrium is "the solution of a system of simultaneous equations representing the demand for goods by consumers, the supply of goods by producers, and the equilibrium condition that supply equal[s] demand on every market" (Arrow and Debreu, 1954, p.265).

The driving factors for reaching an equilibrium are the following three assumptions on the behavior of the producers and consumers in the model: First, each consumer, taking prices as given, chooses a bundle of goods whose cost does not exceed its income and maximizes its utility. Second, producers maximize their profits (or minimize their costs), given their production technology. Third, supply should at least cover demand in each market. These three conditions, together with the requirement that prices should be non-negative constitute a general equilibrium.

The following sections describe the consumer and producer behavior (Sections 3.1 and 3.2, respectively), and the closure of the model with the rest of the world (Section 3.3). The clearing of the markets is described in Section 3.4. The model is setup and solved as a mixed-complementarity problem (MCP). This setup is explained in Section 3.5. Finally, Section 3.6 describes the treatment of time in the model.

3.1 Consumer behavior in the model

The consumers in the model, a representative agent (or several household groups) and the government, maximize their welfare in the form of a hierarchical constant-elasticity of substitution (CES) utility function (see Figure 1). At each nest, the responsiveness to relative changes in the prices of the goods in the nest is defined by the substitution elasticity. The goods are perfect substitutes when the substitution elasticity approaches infinity and perfect complements when it approaches zero. At the lowest level of the hierarchy, the consumer decides on the composition of a bundle of non-energy and energy goods. These two bundles build a composite consumption good that, at the next level, is combined with leisure. At the top level, the composite of consumption and leisure is combined with savings to a measure for welfare.

The utility function as shown in Figure 1 is given by:[3]

$$U = \left[\theta^{cls} \left[\left[\theta^{cl} C^{\rho^{cl}} + (1-\theta^{cl}) LS^{\rho^{cl}} \right]^{1/\rho^{cl}} \right]^{\rho^{cls}} + (1-\theta^{cls}) S^{\rho^{cls}} \right]^{1/\rho^{cls}}, \qquad (1)$$

where LS is the demand for leisure and S the amount of savings. The composite consumption good is defined over the available consumer goods categorized according to the divisions of the Classification of the Purposes of Non-Profit Institutions Serving Households (United Nations, 1999). These goods are listed in Table 4. The substitution and value share parameters are denoted with ρ and θ. The superscripts of the elasticities indicate the nest. The variables and parameters of the equations are described in Table 5. The aggregated consumption (C) good of non-energy (NE) and energy goods (E) is given by:

$$C = \left(\theta^{c} \left[\left(\sum_{ne} \theta^{cne}_{ne} NE^{\rho_{cne}}_{ne} \right)^{1/\rho^{cne}} \right]^{\rho^{c}} + (1-\theta^{c}) \left[\left(\sum_{e} \theta^{ce}_{e} E^{\rho_{ce}}_{e} \right)^{1/\rho^{ce}} \right]^{\rho^{c}} \right)^{1/\rho^{c}}, \qquad (2)$$

[2]The Walras system can be found in the English translation of the original work Walras (2014).
[3]Note that variables are always written in capitals and parameters in small letters.

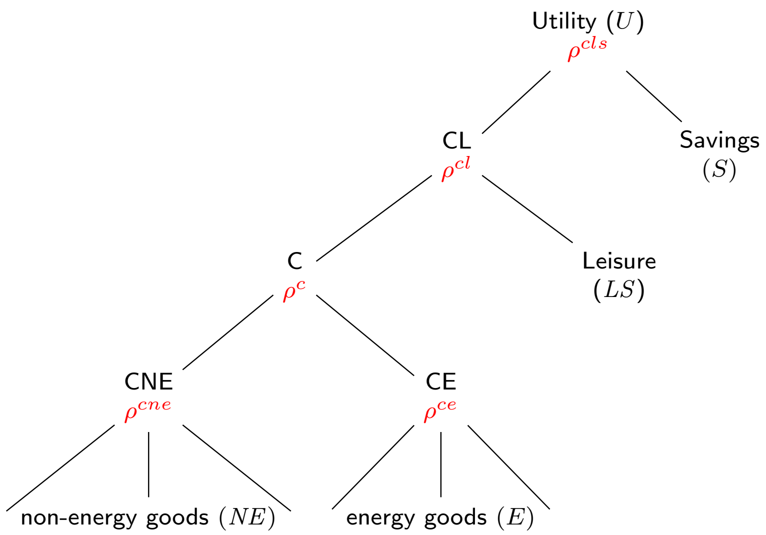


Figure 1: Utility function with substitution parameters (ρ) for each nest.

Table 4: Consumer goods in the model.

COICOP	Description	HABE
C01	Food and non-alcoholic beverages	A51
C02	Alcoholic beverages, tobacco, and narcotics	A52
C03	Clothing and footwear	A56
C04	Housing, water, gas, electricity, and other fuels	A57
C05	Furnishings, household equipment and routine maintenance of the house	A58
C06	Health	A61
C07	Transport	A62
C08	Communication	A63
C09	Recreation and culture	A66
C10	Education	A67
C11	Restaurants and hotels	A53
C12	Miscellaneous goods and services	A68

Table 5: Variables and parameters of the utility functions.

Variable		Parameter	
U	Utility	θ^{cls}	value share of consumption-leisure composite
C	Total consumption	θ^{cl}	subst. parameter of leisure-consumption nest
LS	Leisure	ρ^{cls}	subst. parameter top nest
S	Savings	ρ^{ce}	subst. parameter of energy goods
CNE	Non-energy consumption goods	θ^{cne}	cost share of non-energy goods
CE_e	Consumption of energy good e	θ^{ce}_e	cost share of energy good e
TR	Transfers	θ^{cne}	subst. parameter between non-energy goods

An important aspect of the utility function is the choice of elasticities of substitution. These elasticities measure the percentage change in the ration of two inputs to a percentage change in the ratio of their prices. In empirical work with CGE models, the elasticities are mostly taken from econometric studies. Table 6 shows the values for the substitution elasticities[4] as well as the studies they are taken from.

[4]The substitution parameter and the substitution elasticity are related as follows: $\rho = (\sigma - 1)/\sigma$ and $\sigma = 1/(1-\rho)$.

Table 6: Utility function: Values for the substitution elasticities and their source.

Parameter	Value or range	Source
σ^{cls}	0.28	Havránek (2015)
σ^{cl}	0.7	own calculations based on Jäntti, Pirttilä, and Selin (2015)
σ^{c}	0.9	own assumption
σ^{ce}	0.5	Papageorgiou, Saam, and Schulte (2017)
σ^{cne}	0.9	own assumption

The income for the representative agent (RA) is defined by:

$$I^{RA} = w(\overline{L} - LS) + r\overline{K} + TR - T^{RA}, \tag{3}$$

where $\overline{L}$ is the time endowment, r the rental price of capital endowment K. The income for the government is given as:

$$I^{Gov} = T^{RA} + T^{Prod} - TR \tag{4}$$

The labor endowment of the government (and therefore its leisure demand in the utility function) is zero. The behavior of the representative consumer and the government in the model is now explicitly described by the maximization of the utility function (Equations (1) and (2)) subject to their respective income constraints (Equations (3) and (4)).

The database used for GemEl is the Swiss IOT-Energy of the year 2014 (Nathani et al., 2019). This table provides a detailed description of an economy's circular flow of goods and services. GemEl allows to disaggregate the representative household from the IOT-Energy in 14 separate household groups according to their income and being retired or not (10 working and 4 retired groups). This disaggregation is based on data from the Swiss HBS (Bundesamt für Statistik, 2019a). The HBS is conducted yearly and collects all income and expenditures. Due to the rather small annual HBS sample size (around 3000 households), tables for subgroups can only be based on a pooled sample of at least three years. We use the data for the years 2012, 2013, and 2014. Figure 2 taken from Bundesamt für Statistik (2019a) shows the most important income and expenditures of households in Switzerland.

Every household has a weight that secures that the total sample of around 10'000 households is a good representation of the actual households in Switzerland. If we aggregate using the household weights, we found a discrepancy between the consumer expenditure and income with the numbers in the IOT-Energy 2014. We reconciled the data to get a close match. In a first step, we use the HBS to calculate the total income and expenditure for all households in Switzerland (column "HBS" in Table 7) and compare these figures with the respective figures in the IOT-Energy, the national accounts, and other statistics (column "target"). The table shows that there are greater differences in some of the consumer goods, taxes on income, and capital income. The discrepancy in consumer good C02 (Alcoholic beverages, tobacco, and narcotics) is a typical result in HBSs as households tend to under-report these items. Health expenditure is treated differently at the macroeconomic level and often leads to big discrepancies. In a second step, the household data is scaled to the IOT-Energy values.

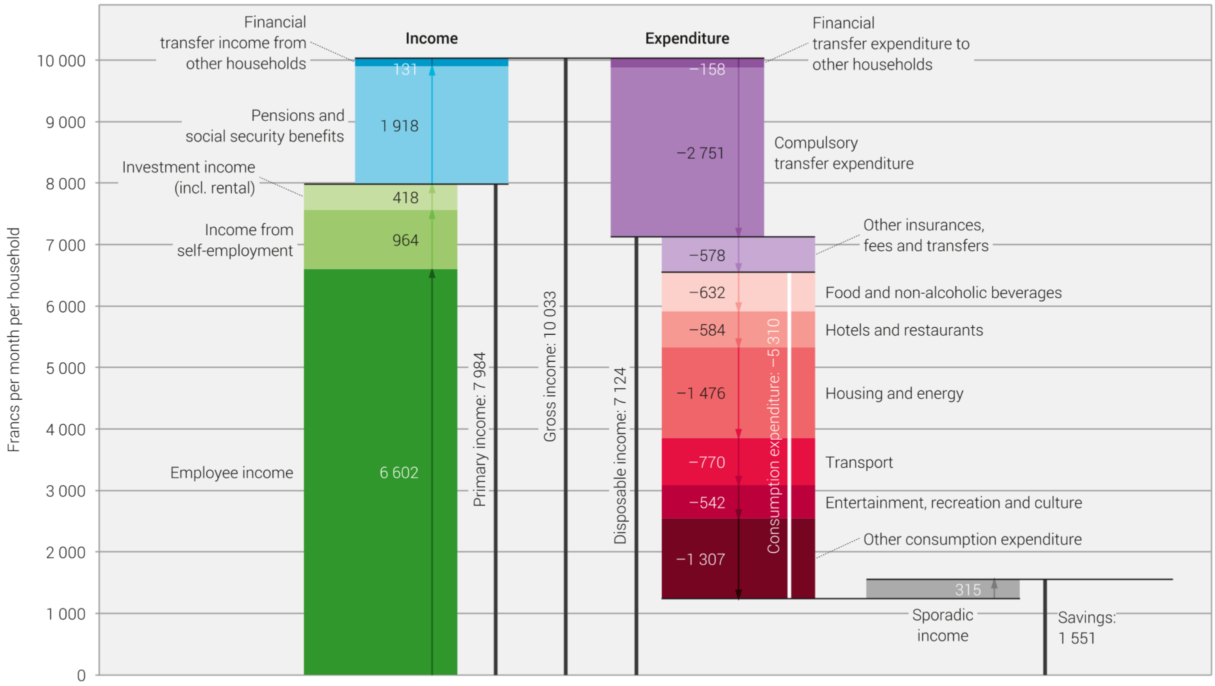


Figure 2: Average income and expenditure of households in Switzerland for the year 2016.(Bundesamt für Statistik, 2019a)

Table 7: Comparison of macro values of household expenditure and income in million CHF.

Code	Description	Source for target	Target	HBS	Difference	Factor
C01	Food and non-alcoholic beverages	IOT2014 [1]	-29'633	-26'985	-2'649	1.1
C02	Alcoholic beverages, tobacco, and narcotics	IOT2014	-8'725	-4'445	-4'279	2.0
C03	Clothing and footwear	IOT2014	-10'293	-9'502	-791	1.1
C04	Housing, water, gas, electricity, and other fuels	IOT2014	-80'983	-63'172	-17'811	1.3
C05	Furnishings, household equipment and routine maintenance of the house	IOT2014	-12'318	-11'586	-733	1.1
C06	Health	IOT2014	-59'417	-11'077	-48'340	5.4
C07	Transport	IOT2014	-27'767	-33'839	6'072	0.8
C08	Communication	IOT2014	-8'087	-7'816	-270	1.0
C09	Recreation and culture	IOT2014	-26'003	-25'450	-553	1.0
C10	Education	IOT2014	-2'060	-1'878	-182	1.1
C11	Restaurants and hotels	IOT2014	-21'995	-23'512	1'517	0.9
C12	Miscellaneous goods and services	IOT2014	-38'144	-10'548	-27'596	3.6
Lab	Labor income	IOT2014	325'381	319'468	5'913	1.0
Cap	Capital income	VGR: S14-D.4 [2]	69'230	33'396	35'833	2.1
IncTax	Taxes on income	VGR: S14-D.5	-68'555	-49'774	-18'781	1.4
Labtax	Social security contributions	BSV [3]	-42'521	-41'919	-602	1.0
Savings	Savings	VGR: S14-B.9	-77'569	-59'271	-18'298	1.3

[1] Input-output table 2014 (Nathani et al., 2019).
[2] VGR: National income accounts (Bundesamt für Statistik, 2019b)
[3] BSV: Federal Office of Social Insurance (Federal Social Insurance Office, 2019)

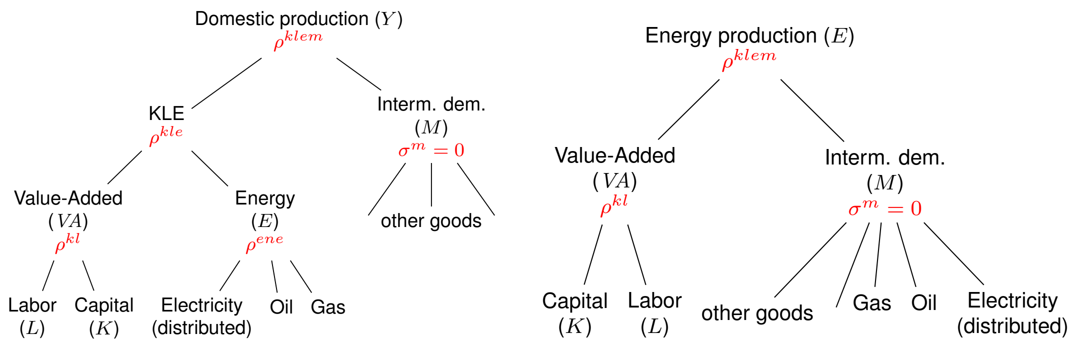
3.2 Producer behavior in the model

GemEI contains over 70 sectors (see Table 8) taken from the Swiss IOT-Energy. Each sector is treated in the model as a producer.

Table 8: Sectors in GemEI.

CPA-Code	Names	CPA-Code	Names
01	Agriculture, hunting and related service activities	40g	Gas supply
02	Forestry, logging and related service activities	41	Collection, purification and distribution of water
05	Fishing, fish farming and related service activities	45	Construction
10-14	Mining and quarrying	50	Sale, maintenance and repair of motor vehicles
15-16	Manufacture of food products, beverages and tobacco	51-52	Wholesale and retail trade
17	Manufacture of textiles	55	Hotels and restaurants
18	Manufacture of wearing apparel, dressing and dyeing of fur	60a	Passenger rail transport
19	Leather and footwear	60b	Eisenbahngüterverkehr
20	Manufacture of wood	60c	Rail infrastructure
21	Manufacture of pulp and paper+C77	60d	Other scheduled passenger land transport
22	Publishing, printing	60e	Taxi operation, Other land passenger transport
23a	Manufacture of coke, refined petroleum products	60f	Freight transport by road
23b	Manufacture of nuclear fuel	60g	Transport via pipelines
24	Chemical industry	61	Water transport
25	Manufacture of rubber and plastic products	62	Air transport
26	Manufacture of other non-metallic mineral products	63a	Water transport infrastructure
27	Manufacture of basics metal	63	Air transport infrastructure / Airports
28	Manufacture of fabricated metal products	63c	Other supporting and auxiliary transport activities; activities of travel agencies
29	Manufacture of machinery and equipment	64	Post and telecommunications
30-31	Manufacture of office and electrical machinery and computers	65	Financial intermediation, except insurance and pension funding (includes also part of NOGA 67)
32	Manufacture of communication equipment	66	Insurance and pension funding, except compulsory social security (includes also part of NOGA 67)
33	Manufacture of medical and optical instruments, watches	70, 97	Real estate (incl. renting by private households)
34	Manufacture of motor vehicles	71, 74	Other business activities
35	Manufacture of other transport equipment	72	Informatics
36	Manufacture of furniture, manufacturing	73	Research and development
37	Recycling	75a	Road infrastructure
40a	Running hydro power plants	75b	Other public administration and defence; compulsory social security
40b	Storage hydro power plants	80	Education
40c	Nuclear power plants	85	Health and social work
40d1	Public power plants (incl. combined heat and power (CHP)) based on fossil fuels	90a	Electricity generation in municiple solid waste (MSW) incineration plants
40d2	Wood based power plants (incl. CHP)	90	Heat generation in MSW incineration plants
40d3	Wind power and photovoltaic (PV) plants	90c	Other waste treatment
40e	Electricity distribution and trade	91-92	Recreational, cultural and sporting activities
40f	Public heat supply	93-95	Private households with employed persons, other service act.

The behavior of each producer is given by the maximization of profits defined as valued output minus the costs of the inputs. In the case of perfect competition, the producer takes the prices of outputs and inputs as given. The production technology is formulated as a nested CES function as shown in Figure 3. We make a distinction between non-energy and energy sectors. In the non-energy sectors, substitution between energy and value-added (capital and labor) is allowed. In the energy sectors, the input of energy fuels is treated as a complementary input to value-added and other inputs to keep inputs and outputs of energy consistent.



(a) Sectors (without energy sectors) (b) Energy production sector

Figure 3: Illustration of domestic production function

We follow van der Werf (2008) in the choice of the substitution possibilities between capital (K), labor (L), energy (E) and intermediate demand (M). He estimates and compares the substitution elasticities of six industrial sectors for several nesting structures (*KE-L, KL-E, KLE*) and finds the highest statistical significance for the elasticities of the *KL-E*-structure. The substitution elasticity in the intermediate nest (σ^m) is set to 0, which is common practice in applied CGE work.[5]

All producers maximize their profits, which is defined as the difference between their sales and costs, given their production function. The production function for the non-energy sectors can be written as:

$$Y_i = \left[\theta_i^{kle}\left[\left[\theta_i^{va}\, VA^{\rho_i^{kle}} + (1-\theta_i^{va})EC^{\rho_i^{kle}}\right]^{1/\rho_i^{kle}}\right]^{\rho_i^{klem}} + (1-\theta_i^{kle})\left(\min_j X_{ji}\right)^{\rho_i^{klem}}\right]^{1/\rho_i^{klem}} . \quad (5)$$

The description of the variables and parameters in Equations (5)-(7) can be found in Table 9.

Table 9: Variables and parameters of the production functions (5)-(7).

Variable		**Parameter**	
Y_i	output	θ_i^{kle}	value share of the composite of KLE
VA_i	value-added	θ_i^{va}	value share of value-added
EC_i	energy composite	ρ_i^{kle}	subst. parameter for KLE nest
X_{ji}	intermediate demand of sector i	ρ_i^{klem}	subst. parameter of top nest
K_i	capital services	θ^k	share of capital costs in VA
L_i	labor	ρ^{kl}	subst. parameter for KL nest
E_{ei}	energy good input e	θ_{ei}^{ene}	cost share of energy good e
ELE_t	Technology t producing electricity	ρ_i^{ene}	subst. parameter of energy nest

The value-added subnest of the production function is given by:

$$VA_i = \left[\theta_i^k K_i^{\rho_i^{kl}} + (1-\theta_i^k)L_i^{\rho_i^{kl}}\right]^{1/\rho_i^{kl}} , \quad (6)$$

[5] A substitution elasticity of zero implies complementary goods: cars need four wheels. However, one reason for setting this value to zero, was the reduction of the complexity of the model in times when computer power was an issue.

and the composite good of energy inputs (EC) is defined as:

$$EC_i = \left[\sum_e \theta_{ei}^{ene} E_{ei}^{\rho^{ene}} \right]^{1/\rho^{ene}} . \tag{7}$$

In the stand-alone version of the model, electricity can be produced using several technologies (nuclear, hydro, solar, PV, etc.). Each technology t is modeled as a Leontief-function:

$$ELE_t = \min \left(L_t, K_t, E_{et}, X_{it} \right) \tag{8}$$

The produced electricity serves as input in the distribution sector. The relative costs of the technologies and the available capacity determine the production mix. Table 10 contains the values or range of the chosen sectoral elasticities.

Table 10: Domestic production and Armington elasticities.

Parameter	Value or range	Source
σ_i^{klem}	0.11 - 1.15	Koesler and Schymura (2015)
σ_i^{kle}	0.09 - 1.27	Koesler and Schymura (2015)
σ_i^{kl}	0.06 - 3.36	Koesler and Schymura (2015)
σ^{ene}	0.5	Papageorgiou, Saam, and Schulte (2017)
σ^{m}	0	common practice in CGE modeling

The producer behavior can now explicitly be described as the maximization of profits given the production functions as defined in Equations (5)-(8).

3.3 International trade

The original general equilibrium formulation is based on a closed economy without any flows coming in from or going out to other countries or regions. In a single-country model like GemEl, sectoral output is transformed into goods produced for the domestic market and exports (see Figure 4). Goods for the domestic market are a composite of imports and domestically produced goods, the so-called Armington good. The domestically produced good is split in domestically supplied goods and exports.

The producer maximizes its profit given the transformation function:

$$\max \Pi_i = P_i^E EX_i + P_i^D D_i - P_i^A A_i \tag{9}$$

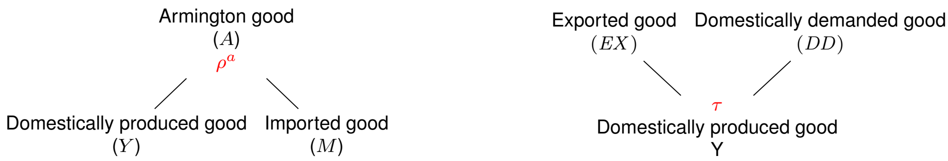
subject to the transformation technology:

$$\left[\theta_i^E EX_i^{\tau} + (1 - \theta_i^E) DD_i^{\psi} \right]^{1/\tau_i} = Y_i. \tag{10}$$

where ψ is the transformation elasticity. The description of the variables and parameters in Equations (9)-(13) can be found in Table 11.

Imports in the model are seen as imperfect substitutes for similar domestically produced goods to allow for cross hauling (importing and exporting the same kind of good). Armington (1969) suggested to replace the domestic consumption by an (Armington) function which converts imported and domestically produced goods into a composite good (see also Figure 4) defined as:

$$A_i = \left[\theta_i^d Y_i \rho_i^a + (1 - \theta_i^D) M_i^{\rho^A} \right]^{1/\rho^{a_i}} \tag{11}$$



(a) Imports (b) Exports

Figure 4: Illustration of the treatment of imports (Armington) and exports.

Table 11: Variables and parameters of the international trade functions.

Variable		**Parameter**	
A	Armington good	ρ	Armington substitution parameter
DD	Domestically demanded good	τ	Transformation parameter between EX and DD
EX	Exports	θ_i^E	share parameter exports
M	Imports	Y	Domestic production

The similarity between imported and domestic goods is measured by the substitution parameter ρ^a. [6] There is no agreement in the literature on the correct value of the sectoral substitution and transformation elasticities (see, for example, Hillberry and Hummels, 2013). Table 12 contains the values or range of the chosen elasticities.

Table 12: International trade and Armington elasticities.

Parameter	**Value or range**	**Source of σ^A and τ**
σ^A	1.2 - 8.0	Own calculations based on Imbs and Méjean (2010)
τ	1.3 - 8.0	and Lofgren and Cicowiez (2018)

We treat Switzerland as a small, open economy, meaning that Switzerland can not influence the world market prices for goods and services. The domestic prices (P) for exports (E) and imports (M) for each good (i) are given by:

$$P_i^e = PFX \cdot \overline{P}_i^{w_e} \quad and \quad P_i^m = PFX \cdot \overline{P}_i^{w_m}, \tag{12}$$

where PFX is the exchange rate, and $\overline{P}^w$ is the given world market price in foreign currency. In the last forty years, except for 1981 and 2008, Switzerland faced a current account surplus.[7] We assume that the surplus is fixed leading to the following additional constraint:

$$\sum_i P_i^e \cdot EX_i + \overline{CA} = \sum_i P_i^m \cdot M_i, \tag{13}$$

where $\overline{CA}$ is the level of the current account surplus, E_i is the quantity of each exported good and M_i is the quantity of each imported good.

In CGE modeling, the investments are treated as Leontief production functions that generate a homogeneous investment good INV.

$$INV_t = \min\left(X_i^{INV}\right). \tag{14}$$

[6] The substitution elasticity σ^A is given by $1/(1-\rho^A)$.

[7] See https://tradingeconomics.com/switzerland/current-account, visited March 9, 2018.

3.4 Market clearing

The third set of conditions for a general equilibrium demands that supply should cover demand in each market (note that this also includes the case of excess supply resulting in a zero price). GemEl contains market clearing conditions for the factors (labor, capital), and produced goods (Armington good, domestically produced good, investment good).

The market-clearing conditions for the factor markets (labor L and capital K) are given by:

$$\sum_i L_i = \overline{L} - LS \text{ and } \sum_i K_i = \overline{K}, \tag{15}$$

while the market-clearing conditions for the domestically produced and the Armington goods are given by:

$$Y_i = D_i + E_i, \text{ and } A_i = \sum_j X_{ij} + X_i^c + X_i^{inv}. \tag{16}$$

Where X_i^C is the consumption demand for good i, and X_i^{inv} is the demand for good i in the investment function. Additionally, the market clearing for the investment good is given by the savings-investment equality:

$$\frac{S}{P^{inv}} = INV. \tag{17}$$

Lastly, the market clearing function for the utility goods of the representative agent (RA) and the government (Gov) is given by:

$$U = \frac{I^{RA}}{PU}, \text{ and } UG = \frac{I^{Gov}}{PUG}. \tag{18}$$

3.5 The mixed-complemetarity format

GemEl is set up and solved as a MCP. Mathiesen (1985) showed that the three Arrow-Debreu conditions for a general equilibrium as discussed above can be formulated as a MCP problem. The MCP format is a special case of a variational inequality problem in which all the variables lie in the positive orthant (see Facchinei and Pang, 2003). The MCP format suits itself for solving general equilibrium models. As Mathiesen (1985, p. 1226) writes, although the first-order optimality conditions of a mathematical programming model also satisfy a complementarity problem (CP) problem, there may be no optimization problem for a general equilibrium model that leads to this CP problem (the so-called integrability-problem, Samuelson, 1950). This can happen if, for example, the model contains several households with distinct endowments and preferences, or if there are ad-valorem taxes or constraints on prices.

A CP can be described as a system of (non-)linear constraints where the system variables are linked to the constraints with complementarity conditions (Ferris and Munson, 2014). More formally, given a function $F : \mathbb{R}^n \to \mathbb{R}^n$, lower bounds $l \in \{\mathbb{R} \cup -\infty\}^n$ and upper bounds $u \in \{\mathbb{R} \cup \infty\}^n$, we try to find $x \in \mathbb{R}^n$ such that precisely one of the following holds for each $i \in 1, \dots, n$:

$$\begin{aligned} F_i(x_i) = l_i \quad &\text{and} \quad F_i(x_i) \geq 0, \quad \text{or} \\ F_i(x_i) = u_i \quad &\text{and} \quad F_i(x_i) \leq 0, \quad \text{or} \\ l_i < x_i < u_i \quad &\text{and} \quad F_i(x_i) = 0 \end{aligned}$$

This means that the variable x_i is at one of its bounds or the linked function is equal to zero.

In the MCP, we not only have inequalities with complementary non-negative variables but also equations where the associated variables are free. The complementarity conditions can then be written as:

$$F_i(x_i, x_j) \geq 0, \quad x_i \geq 0, \quad x_i F_i(x) = 0,$$
$$F_j(x_i, x_j) = 0, \quad x_j \text{ free},$$

where we partition the set n into the sets i and j.

Often the following shorthand notation is used, where the perpendicular symbol ($\perp$) indicates the complementarity slackness between the constraint and the variable:

$$0 \geq F(x) \perp x \geq 0. \tag{19}$$

Complementarity models can be used for solving linear, quadratic and nonlinear programs by writing the Karush-Kuhn-Tucker optimality conditions. In the case of minimizing a function $f(x)$, where $x \in \mathbb{R}^{+}$, the first-order condition is given by:

$$\frac{\partial f}{\partial x} \geq 0, \quad x \geq 0. \tag{20}$$

If x is at its lower bound, we must have that the function is increasing in x. If we have an interior solution, the derivative must be equal to zero. Combining these two pieces of information, we get the mixed complementarity formulation:

$$\frac{\partial f}{\partial x} \leq 0, \quad x \geq 0, \quad x\frac{\partial f}{\partial x} = 0. \tag{21}$$

As the CP can often be formulated using the optimality conditions of the original problem, it is easy to write down the model equations. However, there is not always an optimization problem that corresponds to the complementarity conditions. This means that a MCP formulation allows us to solve a wider class of problems.

Complementary models have been used for expressing a variety of economic equilibrium models for both markets and games, where the underlying problem cannot be written down as a single optimization problem or if no equivalent optimization problem exists, for example, due to non-integrability conditions.[8] Many examples in MCP format can be found in Ferris and Munson (2014), Rutherford (1995), and Dirkse and Ferris (1995). The development of the complementarity modeling format was motivated by theoretical and practical developments in algorithms for nonlinear CPs and variational inequalities. The most recent techniques are based on ideas from interior-point algorithms for linear programming (Kojima et al., 1991). Computational evidence suggests that algorithms for solving MCPs are relatively reliable and efficient, particularly for models that are not natural optimization problems. A survey of developments in the theory and applications of these methods is provided by Harker and Pang (1990).

Mathiesen's MCP version of the CGE model is formulated as a nonlinear system of (weak) inequalities and equalities corresponding to the three classes of equilibrium conditions associated with the Arrow-Debreu general equilibrium. The fundamental unknowns of the system are three vectors consisting of non-negative prices (for commodities and factors), activity levels (production and utility) and household incomes. In equilibrium, each of these variables is linked to one of the inequalities or equalities. The three classes are:

1. The **zero-profit conditions** (more precise, the non-positive profit conditions). In this class the variable complementary to the equation is the activity level: If a sector in equilibrium makes a negative profit, the activity level will be zero; if the profit is zero, the activity level will be positive. Note, that because of the assumption of perfect competition in equilibrium no (excess) profits will exist: Positive profits would lead to new entrants driving the price and, therefore, the profits to zero. The zero-profit functions can be derived from the maximization or, in case of the producers of the dual cost minimization problems.

[8] See the paper on this topic by the famous economist Samuelson (1950).

We use the calibrated share form of the CES function (see Rutherford (1998) and Appendix A) to write down the zero-profit condition for the utility function:

$$P^U \leq \left[\theta^{cls}\left(\frac{P^{inv}}{\overline{P}^{inv}}\right)^{1-\sigma^{cls}} + (1-\theta^{cls})\left[\left(\frac{P^{cls}}{\overline{P}^{cls}}\right)^{\frac{1}{1-\sigma^{cl}}}\right]^{1-\sigma^{cls}}\right]^{\frac{1}{1-\sigma^{cls}}} \perp U$$

$$\text{where } P^z = \left[\theta^{cl}\left(\frac{P^{ls}}{\overline{P}^{ls}}\right)^{1-\sigma^{cl}} + (1-\theta^{cl})\left(\frac{P^{C}}{\overline{P}^{c}}\right)^{1-\sigma^{cl}}\right]$$

$$\text{and } P^c = \left[\theta^{c}\left(\frac{P^{ne}}{\overline{P}^{ne}}\right)^{1-\sigma^{c}} + (1-\theta^{c})\left(\frac{P^{e}}{\overline{P}^{e}}\right)^{1-\sigma^{c}}\right]^{\frac{1}{1-\sigma^{c}}} \tag{22}$$

$$\text{where } P^{ne} = \left[\sum_{sne}\theta^{cne}_{sne}\left(\frac{P^{A}_{sne}}{\overline{P}^{A}_{sne}}\right)^{1-\sigma^{cne}}\right]^{\frac{1}{1-\sigma^{cne}}}$$

$$\text{and } P^e = \left[\sum_{e}\theta^{ce}_{e}\left(\frac{P^{A}_{e}}{\overline{P}^{A}_{e}}\right)^{1-\sigma^{ce}}\right]^{1/1-\sigma^{ce}}$$

Table 13 contains the description of variables and parameters not mentioned before.

Table 13: Variables and parameters of the zero-profit condition.

Variable		**Parameter**	
Y	output	θ^{kle}	value share of the composite of KLE
VA	value-added	θ^{va}	value share of value-added
EC	energy composite	ρ^{kle}	subst. parameter for KLE nest
X_i	intermediate demand of sector i	ρ^{klem}	subst. parameter of top nest
K	capital services	θ^{k}	share of capital costs in VA
L	labor	ρ^{kl}	subst. parameter for KL nest
E_e	energy good e	θ^{ene}_{e}	cost share of energy good e
ELE_t	Technology t producing electricity	ρ^{ene}	subst. parameter of energy nest

Using the calibrated share form, it is straightforward to write down the other zero-profit conditions. We refrain from writing down these equations in the extensive form and use a condensed form. The zero-profit function for the government utility, the domestic non-energy and energy sectors, the Armington sectors as well as the investment sector is given by:

$$\text{Government utility:} \quad -\Pi^{G} \geq 0 \perp UG \tag{23}$$

$$\text{Non-energy domestic production:} \quad -\Pi^{D}_{ne} \geq 0 \perp Y_{ne} \tag{24}$$

$$\text{Energy domestic production:} \quad -\Pi^{D}_{e} \geq 0 \perp Y_{e} \tag{25}$$

$$\text{Armington sector:} \quad -\Pi^{A}_{i} \geq 0 \perp A_{i} \tag{26}$$

$$\text{Investment sector:} \quad -\Pi^{inv}_{i} \geq 0 \perp INV_{i}. \tag{27}$$

2. The **market clearing conditions**. These equations are complementary with the prices: Supply minus demand for every commodity should be non-negative. In equilibrium, a positive supply means that the complementary price is zero (the case of a free good); if supply is equal to demand, a positive equilibrium price will be the result. The market clearing conditions can be derived using Shephard's lemma. This lemma states that the conditional demand for an input in production is

equal to the derivative of the cost function with respect to the price of the input (Varian, 1992).

$$A_i = \sum_j \frac{\partial C_i^D}{\partial P_i^A} + \frac{\partial C_i^U}{\partial P_i^A} \tag{28}$$

All other market clearing functions can be derived in the same way (i.e. differentiation of the cost functions into the respective production functions).

3. **Income balance or definition**: This class of equations simplifies the market-clearing conditions as the expression for income in the consumer (RA) or government (Gov) consumption demand functions can be replaced by a single variable (I^{RA} and I^Gov).

3.6 Time in the CGE model

There are several approaches for incorporating time in a CGE model. The above description of the model assumes a static treatment of time. GemEl has two options to incorporate time:

1. It can be solved as a recursive model in which agents do not form consistent expectations of future prices as their decisions are based on the actual information. This kind of model is suitable for the Nexus-e framework as the other modules are static. Using a recursive dynamic framework, GemEl can be easily solved and updated for the next year. The new information can then be send to the other, static modules.
2. It can be solved for other years by extrapolating capital and labor endowments to the expected level in the following years. This is a good option if the total time of solving the framework is taking too much time to solve for each year.

A key input variable for the implementation of recursive dynamics is the gross investment in the previous period. This variable is used to update the available capital stock with the capital movement equation:

$$K_{t+1} = (1 - \delta)K_t + I_t, \tag{29}$$

where the capital in the next period (K_{t+1}) is defined as the depreciated capital in the current period(K_t) plus the actual investments (I_t), where δ is the depreciation rate. If there are multiple households, we assume that every household's savings are equal to total investment times their share of total capital.

To check if the model is correctly calibrated, meaning that it reproduces the data that serve as a starting point, the recursive model is calibrated to a steady-state baseline equilibrium growth path using the fact that on a steady-state growth path all quantities grow with the same growth rate. This means that capital is also growing according to the following equation:

$$K_{t+1} = (1 + \gamma)K_t, \tag{30}$$

where γ is the assumed steady-state growth rate. We can now use Equations (29) and (30) and information from the IOT-Energy to calibrate the model to the given growth path. For Switzerland, we assume a steady-state growth rate of 1.5% (see Table 14 for the growth projections until 2050).

This steady-state growth path does not take into account the changes in the working population. We, therefore, use the projections on the working population (BFS Scenario A-00-2015) to calculate the yearly percentage change (γ_t^{WP}). The total GDP growth rate γ^{GDP} is now given by:

$$\gamma_t^{GDP} = (1 + \gamma^{GDP/Cap})(1 + \gamma_t^{WP}) - 1. \tag{31}$$

The energy demand projections (electricity and fossil fuels) are shown in Table 14 and Figure 5. To reach the given levels, we adjust the technical progress for the energy goods to calibrate demand to the projections from the Energy Perspectives.

Table 14: Assumed projections for Swiss population, GDP and energy demand (Swiss Energy Modelling Platform, 2018).

Parameter	2010	2020	2035	2050	Reference
Population (million)	7.79	8.68	9.8	10.3	BFS Scenario A-00-2015
Working population (million full time equivalents)	3.853	4.31	4.58	4.63	BFS Scenario A-00-2015
GDP potential3 (relative to 2010)	1	1.18	1.43	1.66	Projections from: SECO 2015
Energy demand (relative to 2010)	1	0.937	0.839	0.782	BAU (WWB) scenario from BFE 2050 Energy Perspectives (p. 96)
Electricity demand (relative to 2010)	1	1.05	1.097	1.175	BAU (WWB) scenario from BFE 2050 Energy Perspectives (p. 96)
Fossil energy demand by ETS sectors (relative to 2010)	1	0.858	0.621	0.388	Simlab

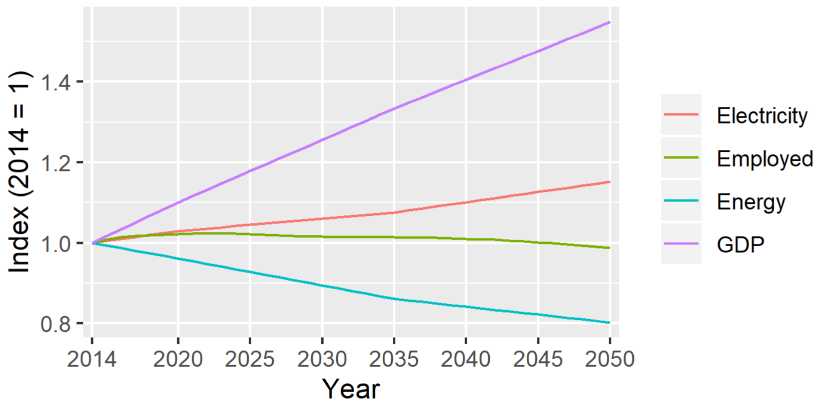


Figure 5: Illustration of projections according to Swiss Energy Modelling Platform (2018)

4 Description of interfaces

The interfaces of GemEl with the other modules is part of the Energy-Economic loop. In this loop, by combining the bottom-up optimizations of investment and operating decisions done in CentIv, DistIv, and eMark with the top-down economic equilibrium in GemEl, Nexus-e is able to account for the response of the economy to the changing infrastructure and investments in the electricity system. Similarly, Nexus-e can also assess how investments and operation of the electricity infrastructure would change in response to major shifts in the Swiss economy. An iterative convergence process is necessary for this loop since the modules cannot be solved simultaneously and each module depends on inputs from the other modules. This convergence process was developed with the goal of improving how top-down economy models and bottom-up energy system models are linked. In Nexus-e, GemEl is the top-down CGE model and the other modules in the Energy-Economic loop are all considered bottom-up energy system models (i.e. CentIv, DistIv, and eMark).

As a starting point, GemEl provides the Investments loop (i.e. CentIv and DistIv) with the yearly electricity demand and the price indices for the generation technologies. CentIv and DistIv calculate the necessary investments in new generation capacities. Then eMark calculates the market dispatch, generator operating costs and electricity imports/exports, and sends them to GemEl along with the investment costs from CentIv and DistIv. GemEl then calculates a new annual demand and new generator price indices and passes these data back to CentIv and DistIv, beginning a new iteration of the Energy-Economic loop. This process continues until the yearly demand quantified by GemEl converges, i.e. the difference between the demand in the current iteration and the demand of the previous iteration (or the base case) is smaller than 2%.

Originally, the convergence procedure was modeled using the methodology taken from Böhringer and Rutherford 2009. This technique uses the equilibrium electricity price and demand from a top-down macroeconomic model to derive a linear approximation of the demand curve. The demand curve is then used to determine the generation mix in the bottom-up model, and total welfare is maximized. This technique works fine for a highly aggregated bottom-up model with a yearly resolution as there is only one demand curve, and top-down and bottom-up models are built by the same modeler. However, if the bottom-up models have an hourly resolution like in Nexus-e, this means that the highly aggregated demand of the top-down model has to be split into hourly load curves magnifying the number of optimization problems by a factor 8’760. Furthermore, since the modules in Nexus-e are developed by different researchers using different software, this methodology is not easily implemented. Therefore, a approach for enabling this top-down and bottom-up connection was derived for Nexus-e that involves a recalibration of the supply/demand equilibrium based on the costs and generation mix of the bottom-up models (CentIv, DistIv, and eMark).

In the bottom-up models, demand is given and supply is calculated endogenously. However, if the cost of the generation mix changes, this will have an impact on demand in GemEl. For example, if the bottom-up models show that replacing the supply of nuclear power leads to a massive investment in expensive technologies, this change will increase the electricity price in Switzerland and lead to a shift of the supply curve in GemEl and will result in a new supply/demand equilibrium. Figure 6 provides an illustration of the interactions between the modules in such a situation. In response to the higher costs of the new generation mix, industries and households will demand less electricity and try to substitute electricity with other energy sources (i.e. the supply curve shifts to the left implying an increase in generator costs). Once calibrated, the new equilibrium will yield the new annual Swiss electricity demand.

The information on investments, costs, imports, exports, and generation mix provided by CentIv, DistIv, and eMark are used in GemEl to recalibrate the generation technologies and distribution sector (see Figure 7) and determine the new electricity demand from the supply/demand equilibrium. For this recalibration, the information from the bottom-up models is used to replace the domestic generation in

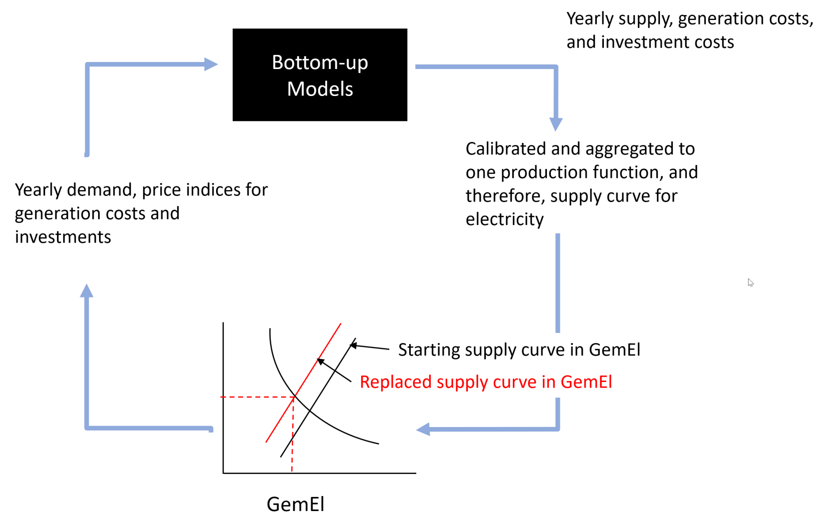


Figure 6: Interface between GemEl and bottom-up models.

GemEl. While the domestic generation is fixed, the exports and imports are used as a starting point but allowed to adjust to the new demand. A change in demand is a reaction to changes in the electricity price caused by the change in generation mix and trade. In GemEl the domestic price of electricity is a mixture of the price of domestic generation and the price of imports. In Figure 7 an example is shown: on the left is the original situation with the costs for the domestic generation mix and the electricity distribution. On the right, the situation is shown after taking into account the results from the bottom-up models (a replacement of low cost nuclear power by more costly renewables). Also, according to the bottom-up models, domestic generation and exports are reduced and imports increased. The recalibration and changes in trade will lead to a shift of the supply curve in the top-down model (in the figure, the supply curve shifts to the left, implying an increase in generation costs) and demand goes down. The shift of the supply curve leads not only to a change in electricity demand but can also lead to a change in the prices of other goods and income of the households. These second-round effects, although smaller, can lead to further changes in the demand. This change in electricity demand could lead to different generator investment decisions, hence the need to connect and iterate these processes.

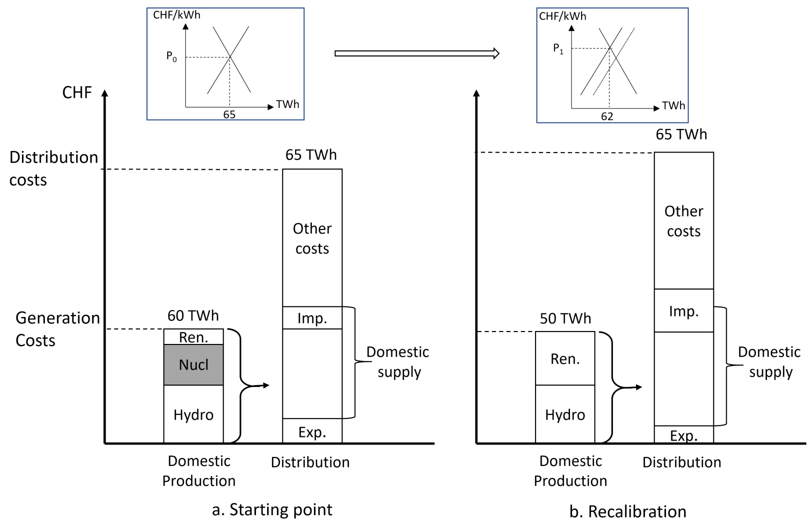


Figure 7: Recalibration of the electricity sector.

The detailed exchanges of data among GemEl and the other modules in the Energy-Economic Loop are given in the following subsections.

4.1 Investments-GemEl interface

The interface from the Investments loop to GemEl passes cost information for all generators, those newly built as well as those already existing, on both the transmission and distribution system levels. This information is mapped to the technology types in GemEl and used to recalibrate the module to reflect the new generation mix and costs. While the interface can be thought of as one process, in fact both CentIv and DistIv have individual functions that are called by GemEl to package and send the appropriate data. Having separate functions allows more flexibility within the framework (e.g. running a simulation without DistIv). Both functions provide to GemEl the same type of data for the generators in their respective modules. Table 15 shows details of the data transferred in the CentIv-GemEl and DistIv-GemEl interfaces.

Table 15: CentIv-GemEl and DistIv-GemEl module interface details.

Variable	Resolution	Unit	Description
Investment cost	annual, by unit type	mill CHF	Investment cost per technology type
Fixed operation and maintenance (OM) cost	annual, by unit type	mill CHF	Fixed OM cost per technology type

4.2 eMark-GemEl interface

The interface from eMark to GemEl passes information on the annual operating costs and generation share by technology type for new and existing generators. This information is mapped to the technologies used in the GemEl module and used to recalibrate GemEl to reflect the new generation mix and costs. Table 16 shows details of the data transferred in this interface.

Table 16: eMark-GemEl module interface details.

Variable	Resolution	Unit	Description
Variable OM cost	annual, by unit type	mill CHF	Variable OM costs per technology type
Generation share	annual, by unit type	megawatt hour (MWh)	Generation per technology type

4.3 GemEl-Investments interface

The GemEl-Investments interface provides feedback from the macroeconomic GemEl module to the bottom-up energy modules CentIv and DistIv about how the economy and consumers respond to the expenses incurred in the electricity system. The feedback is in the form of a change to the annual Swiss demand and a change to the operating and investment costs for generating units. Table 17 shows details of the data transferred in this interface.

Table 17: GemEl-Investments module interface details.

Variable	Resolution	Unit	Description
Total Swiss demand	annual	MWh	Yearly demand in Switzerland
Price index for variable OM costs	annual	–	Change in variable OM costs
Price index for fixed OM costs	annual	–	Change in fixed OM costs
Price index for investment costs	annual	–	Change in investment costs

CentIv compares the adjusted annual Swiss demand to the initial value of the annual demand pulled from the database and calculates their ratio as a scaling factor. The scaling factor is applied as a multiplier to the hourly nodal Swiss load profiles to re-scale them to match the adjusted total from GemEl. Similarly, new price indices provided by GemEl for operating and investment costs are used as a multiplier to reset all existing and candidate variable OM costs, fixed OM costs, and investment costs. CentIv and DistIv will both use the re-scaled load profiles and costs in the next iteration of the Energy-Economic Loop to re-evaluate the optimal investments.

5 Demonstration of results

The demonstration results in this section provide a highlight of the capabilities and insights that GemEl can provide. These results are only for illustrative purposes and are not meant to represent the final results of the Nexus-e simulation framework for any particular scenario.

GemEl can give insights about the changes of economic parameters and it reports on the following indicators and measures (all yearly, for Switzerland):

- Macroeconomic indicators: GDP, total exports, imports, tax revenue
- Sectoral indicators: production, prices, imports, exports
- Welfare measures: Change in welfare, distributional effects
- Energy and environment: CO_2 price of permits, CO_2 tax

A word of caution is necessary as a CGE model is not a projection model: The development of key parameters like the growth rate, change in employment, or technological change are exogenously given and used to calibrate the model to a given growth path and a set of policy measures. This calibrated scenario is usually labeled as the business-as-usual (BAU) scenario. For example, in the BAU scenario, the level of the GDP in 2050 is an assumption and not a result. A CGE model is, therefore, commonly used for comparing different scenarios with the BAU scenario. A certain scenario might result in a higher or lower GDP than in the BAU scenario.

The following figures illustrate the range of results that can be derived from scenario analysis with GemEl. For this illustrative analysis, we compare the baseline scenario without CO_2 taxation with a scenario where the CO_2 taxes that are expected in 2050 are in place. These simulations were done with the stand-alone version of GemEl.

A typical output is a change in producer prices (see Figure 8).

Figure 8: Changes in domestic producer prices.

If we use the disaggregation of the households, we can check if a simulated policy is regressive (hits the lower-income households relatively harder than the richer households) or progressive. Figure 9 shows a regressive policy (the first ten households, numbered EH1 to EH10, are the income deciles of the working households, the last four (RH1-RH4) are the income quartiles of the retired households).

If GemEl is run as stand-alone, it can report the domestic supply of electricity (see Figure 10).

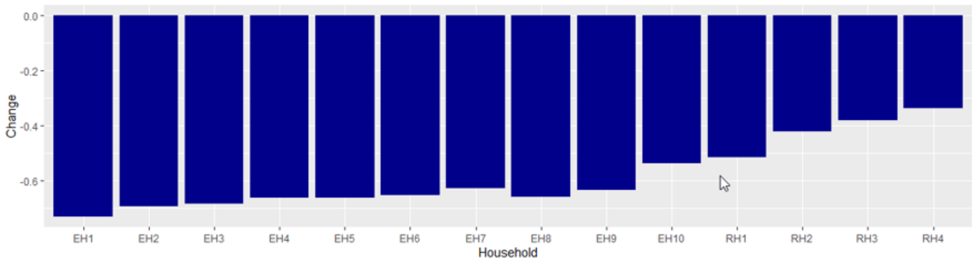


Figure 9: Change in household utility.

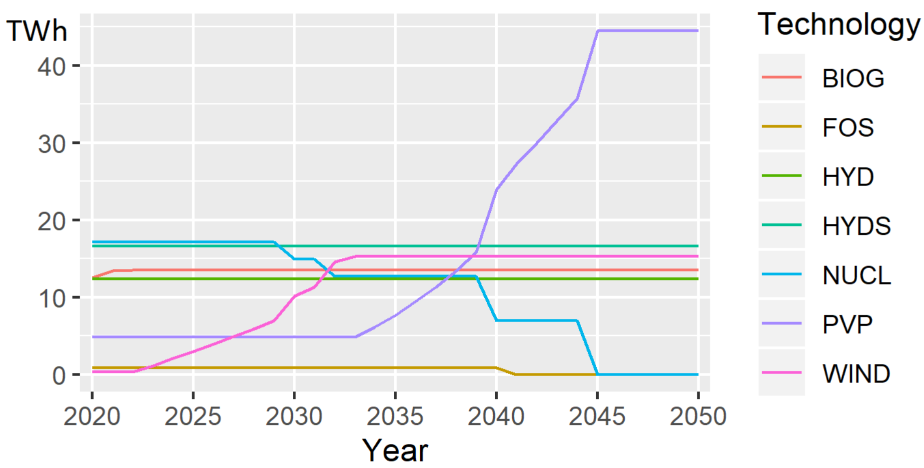


Figure 10: Domestic electricity generation.

6 Publications

The following list describes publications related to the Nexus-e platform:

- A poster presented at the 2018 Conference by the Energy Modeling Platform for Europe (EMP-E) (van Nieuwkoop et al., 2018) provides an overview of the Nexus-s integrated energy systems modeling platform.
- An article published in 2018 in the Energy Strategy Reviews journal (Crespo Granado et al., 2018) provides a thorough review of existing works related to modeling dimensions of the energy transition along with methods employed to combine various model types. The article then presents a proposal for an integrated linking of top-down and bottom-up models to represent: distributed generation and demand, operations of electricity grids, infrastructure investments and generation dispatch, and macroeconomic interactions.

7 References

Aguiar, Angel, Badri Narayanan, and Robert McDougall (2016). “An Overview of the GTAP 9 Data Base”. In: *Journal of Global Economic Analysis* 1.1, pp. 181–208.

Armington, Paul S. (1969). “A Theory of Demand for Products Distinguished by Place of Production”. In: *IMF Staff Papers* 16.1, pp. 159–178.

Arrow, Kenneth J. and Gerard Debreu (1954). “Existence of an Equilibrium for a Competitive Economy”. In: *Econometrica* 22.3, pp. 265–290.

Bernard, Alain and Marc Vielle (Feb. 2008). “GEMINI-E3, a general equilibrium model of international-national interactions between economy, energy and the environment”. In: *Computational Management Science* 5, pp. 173–206.

Böhringer, Christoph and André Müller (2014). “Environmental tax reforms in Switzerland a computable general Equilibrium impact analysis”. In: *Swiss Journal of Economics and Statistics* 150.1, pp. 1–21.

Böhringer, Christoph and Thomas F. Rutherford (2009). “Integrated assessment of energy policies: Decomposing top-down and bottom-up”. In: *Journal of Economic Dynamics and Control* 33.9, pp. 1648–1661.

Bretschger, Lucas and Lin Zhang (2017). “Nuclear Phase-out Under Stringent Climate Policies: A Dynamic Macroeconomic Analysis”. In: *The Energy Journal* 38 (1), pp. 1944–9089.

Bundesamt für Statistik (2015). *Szenarien zur Bevölkerungsentwicklung der Schweiz 2015-2045*. Tech. rep. Bundesamt für Statistik.

— (2019a). *Household Budget Survey 2012-2014*. Data request. Bundesamt für Statistik.

— (2019b). *Sequence of accounts of the total economy and the institutional sectors*. URL: `http://www.bfs.admin.ch/bfs/portal/en/index/themen/04/02/01/key/kontensequenz.Document.78706.xls` (visited on 08/01/2019).

Crespo Granado, Pedro del et al. (2018). “Modelling the energy transition: A nexus of energy system and economic models”. In: *Energy Strategy Reviews* 20, pp. 229–235.

Dirkse, Steven P. and Michael C. Ferris (1995). “Mcplib: a collection of nonlinear mixed complementarity problems”. In: *Optimization Methods and Software* 5.4, pp. 319–345.

Facchinei, Francisco and Jong-Shi Pang (2003). *Finite-Dimensional Variational Inequalities and Complementarity Problems*. Springer.

Federal Office for the Environment (2019). *Table CO2-Statistik-2019-07_D: CO2 statistics: Emissions from thermal and motor fuels*. URL: `https://www.bafu.admin.ch/bafu/en/home/topics/climate/state/data/co2-statistics.html` (visited on 07/26/2019).

Federal Social Insurance Office (2019). *Statistics on occupational benefit plans and the 3rd Pillar*. URL: `https://www.bsv.admin.ch/bsv/en/home/social-insurance/bv/statistik.html` (visited on 08/01/2019).

Ferris, Michael C. and Todd S. Munson (2014). *PATH 4.7*. Gams Corporation. Washington.

Harker, Patrick T. and Jong-Shi Pang (1990). “Finite-dimensional variational inequality and nonlinear complementarity problems: A survey of theory, algorithms and applications”. In: *Mathematical Programming* 48 (1), pp. 161–220.

Havránek, Tomáŝ (Dec. 2015). “Measuring Intertemporal Substitution: The Importance Of Method Choices And Selective Reporting”. In: *Journal of the European Economic Association* 13.6, pp. 1180–1204.

Hillberry, Russell and David Hummels (2013). “Trade Elasticity Parameters for a Computable General Equilibrium Model”. In: ed. by Peter B. Dixon and Dale W. Jorgenson. Elsevier B.V. Chap. Handbook of Computable General Equilibrium, pp. 1213–1269.

Imbs, Jean and Isabelle Méjean (2010). *Trade Elasticities - A Final Report for the European Commission*. Tech. rep. European Commission.

Jäntti, Markus, Jukka Pirttilä, and Hakan Selin (2015). “Estimating labour supply elasticities based on cross-country micro data: A bridge between micro and macro estimates?” In: *Journal of Public Economics* 127. The Nordic Model, pp. 87–99.

Klump, Rainer, Peter McAdam, and Alpo Willman (Feb. 2011). *The normalized CES production function - Theory and empirics*. Research rep. 1294. European Central BANK.

Klump, Rainer and Marianne Saam (2008). “Calibration of normalised CES production functions in dynamic models”. In: *Economics Letters* 99.2, pp. 256–259.

Koesler, Simon and Michael Schymura (2015). “Substitution Elasticities in a Constant Elasticity of Substitution Framework – Empirical Estimates using Nonlinear Least Squares”. In: *Economic Systems Research* 27.1, pp. 101–121.

Kojima, Masakazu et al. (1991). *A unified approach to interior point algorithms for linear complementarity problems*. Lecture Notes in Computer Science 538. Springer-Verlag.

Landis, Florian (Sept. 2019). “Cost distribution and equity of climate policy in Switzerland”. In: *Swiss Journal of Economics and Statistics* 155.1, p. 11.

Landis, Florian et al. (Sept. 2019). “Multi-model comparison of Swiss decarbonization scenarios”. In: *Swiss Journal of Economics and Statistics* 155.1, p. 12.

Lofgren, Hans and Hans Cicowiez (2018). *Linking Armington and CET Elasticities of Substitution and Transformation to Price Elasticities of Import Demand and Export Supply: A Note for CGE Practitioners*. Tech. rep.

Mathiesen, Lars (1985). “Computation of economic equilibria by a sequence of linear complementarity problems”. In: *Economic Equilibrium: Model Formulation and Solution*. Ed. by Alan S. Manne. Berlin, Heidelberg: Springer Berlin Heidelberg, pp. 144–162.

Nathani, C. et al. (2019). *Energie- und verkehrsbezogene Differenzierung der Schweizerischen Input-Output-Tabelle 2014*. Tech. rep. Federal Office of Energy.

Papageorgiou, Chris, Marianne Saam, and Patrick Schulte (2017). “Substitution between Clean and Dirty Energy Inputs: A Macroeconomic Perspective”. In: *The Review of Economics and Statistics* 99.2, pp. 281–290.

Peters, Jeffrey C (2016). “The GTAP-Power Data Base: Disaggregating the Electricity Sector in the GTAP Data Base”. In: *GTAP Journal of Global Economic Analysis* 1.1, pp. 209–250.

Prognos (2012). *Die Energieperspektiven für die Schweiz. Energienachfrage und Elektriziätsproduktion in der Schweiz 2010 - 2050 Ergebnisse der Modellrechnungen für das Energysystem*. Tech. rep. Prognos.

Romer, Paul M. (1990). “Endogenous Technological Change”. In: *Journal of Political Economy* 98.5, S71–S102.

Rutherford, Thomas F. (1995). “Extension of GAMS for complementarity problems arising in applied economic analysis”. In: *Journal of Economic Dynamics and Control* 19.8, pp. 1299–1324.

— (June 1998). *Economic Equilibrium Modeling with GAMS*.

— (2002). *Lecture notes on the Constant Elasticity of Substituion function*. mimeo. University of Colorado.

Samuelson, Paul A. (1950). “The Problem of Integrability in Utility Theory”. In: *Economica*. New Series 17.68, pp. 355–385.

Swiss Energy Modelling Platform (2018). *Harmonized scenario assumptions*. URL: `http://simlab.ethz.ch/1stSemp_sceAssump.php` (visited on 07/01/2019).

Swiss Federal Office of Energy (2019). *Schweizerische Elektrizitätsstatistik 2018*. Tech. rep. Swiss Federal Office of Energy.

Temple, Jonathan (2012). “The calibration of CES production functions”. In: *Journal of Macroeconomics* 34.2, pp. 294–303.

United Nations (1999). *Classifications of Expenditure According to Purpose*. Series M 84. Department of Economic and Social Affairs - Statistics Division.

van der Werf, Edwin (2008). “Production functions for climate policy modeling: An empirical analysis”. In: *Energy Economics* 30.6. Technological Change and the Environment, pp. 2964–2979.

van Nieuwkoop, Renger H. et al. (2018). “Nexus Energy System Platform”. In: *Energy Modelling Platform for Europe (EMP-E) 2018 Conference on Modelling Clean Energy Pathways*. Brussels, Belgium.

Varian, Hall R. (1992). *Microeconomic Analysis*. Norton.

Walras, Léon (2014). *Léon Walras: Elements of Theoretical Economics: Or, The Theory of Social Wealth*. Ed. by Donald A. Walker and Jan van Daal. Cambridge University Press.

Appendices

A Calibrated share form: formulas

The mathematical formulation of the zero-profit and market clearing conditions is based on the calibrated share cost and demand functions (see Table 18).

The calibrated share functions use the substitution elasticity, the benchmark quantities, and prices to calibrate a CES function. For a simple CES function with two inputs x_i and x_j the intuition behind the calibration is shown in Figure 11: The two benchmark demand quantities ($\bar{x}_i$ and $\bar{x}_j$) define on which isoquant of the production function we are. The benchmark prices w_i and w_j define the slope, and the substitution elasticity the curvature of the isoquant.

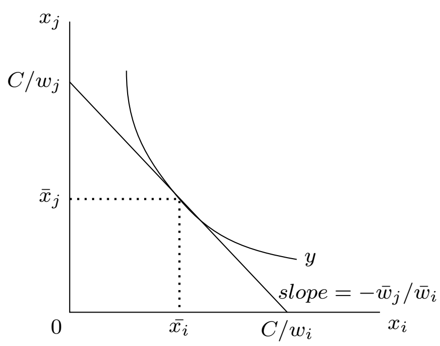


Figure 11: Calibration of the CES function with benchmark values.

The cost and demand function in calibrated-share form are shown in Table 18. Note that the demand and cost functions are easily found by setting the substitution elasticity in the CES functions to zero. The derivation of these functions is shown in Rutherford (2002). More information on the calibrated-share (or normalized) form can be found in Klump and Saam (2008), Temple (2012), and Klump, McAdam, and Willman (2011).

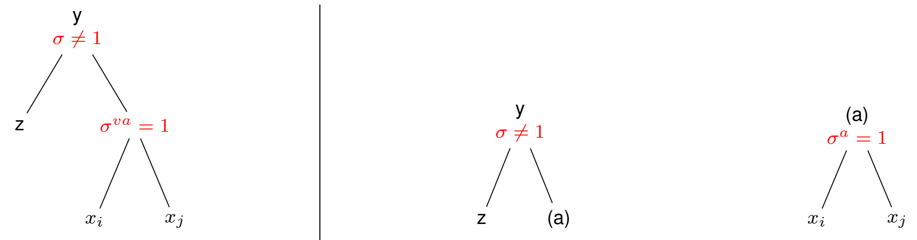
Table 18: Calibrated share formulas for CES and Cobb-Douglas functions.

	CES/Leontief	Cobb-Douglas
Production function	$y(x) = \left(\sum_i \alpha_i x_i^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}$	$y(x) = \prod_i x_i^{\alpha_i}$
Cost function	$c = \bar{c}\left[\sum_i \theta_i \left(\frac{w_i}{\bar{w}_i}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}$	$c = \bar{c}\prod_i \left(\frac{w_i}{\bar{w}_i}\right)^{\theta_i}$
Demand function	$x_i = \bar{x}_i \frac{y}{\bar{y}} \left(\frac{c}{\bar{c}} \frac{\bar{w}_i}{w_i}\right)^{\sigma}$	$x_i = \bar{x}_i \frac{y}{\bar{y}} \left(\frac{c}{\bar{c}} \frac{\bar{w}_i}{w_i}\right)$

In applied CGE work, the multi-level CES function is often used. Deriving the demand and cost functions might, even with the calibration share form, look daunting. However, one can use the calibrated share form of the demand or cost function for a one-level CES or Cobb-Douglas function by splitting the function in a set of one-level functions or by carefully substituting the demand and cost functions of the levels above. The former has the disadvantage that it increases the number of variables in the model (for example, if we have a hierarchical CES function with four levels, the number of variables is multiplied at least by two times the number of nests). In the following, we will show how to use the substitution method for a two-level CES function of the form (see also Figure 12a):

$$y = \left(\alpha_z z^{\frac{\sigma-1}{\sigma}} + (1-\alpha_z) \left[x_i^{\theta} x_j^{1-\theta} \right]^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} \tag{32}$$

The production function consists of an upper CES nest with substitution elasticity σ and a lower Cobb-Douglas nest (substitution elasticity equal to one). We now split the function in a lower and upper nest function (Figure 12b and 12c). The upper nest function produces y using the good z and a composite good "a" out of the two inputs x_i and x_j. In the benchmark, we set prices equal to one and scale output



(a) Production function (b) Upper nest function (c) Lower nest function

Figure 12: Splitting a two-level nested CES production function in two separate single-level production functions.

so that the activity levels are also equal. The demand function for the composite good a in the upper nest function is given by (using the CES calibrated share demand function from Table 18):

$$a = y\bar{a} \left(\frac{p^y}{p^a} \right)^{\sigma} = y\bar{a} \left(\frac{p^y}{w_i^{\theta} w_j^{1-\theta}} \right)^{\sigma} \tag{33}$$

Note that all the benchmark prices and the output level $\bar{y}$ drop out because they are equal to one. The demand for the good x_i in the lower nest function is given by:

$$x_i = \bar{x}_i \frac{a}{\bar{a}} \left(\frac{w_i^{\theta} w_j^{1-\theta}}{w_i} \right)^{\theta} \tag{34}$$

Substituting a from the upper-level demand function in the lower nest demand functions gives the demand function for the good x_i in the production function y.

$$x_i = y\bar{x}_i \left(\frac{p^y}{w_i^{\theta} w_j^{1-\theta}} \right)^{\sigma} \left(\frac{w_i^{\theta} w_j^{1-\theta}}{w_i} \right)^{\theta} \tag{35}$$